ROYAL INSTITUTE
OF TECHNOLOGY

Theoretical Geodesy

Huaan Fan

ISBN 91-7170-646-1

Royal Institute of Technology
Division of Geodesy
100 44 Stockholm
Sweden

September 2008

Theoretical Geodesy

Huaan Fan, PhD

Royal Institute of Technology (KTH)
Division of Geodesy
Teknikringen 72, 100 44 Stockholm, Sweden

Phone: +46 8 7907340
Fax: +46 8 7907343
E-mail: hfan@infra.kth.se
URL: http://www.infra.kth.se/impgg/

ISBN 91-7170-646-1

Geodesy Report 2016
TRITA-GEOFO 2000:23

Printed by **Universitetsservice AB**, Drottning Kristinas väg 53 B, Stockholm
October 2000, August 2001, August 2003, August 2004, August 2005, August 2006, August 2007, September 2008.

” This must be left to the geodesists.
I have no doubt that in the not too distant future
we will be successful in making a precise measurement
of the drift of North America relative to Europe. ”

—— Alfred Wegener, 1929

Preface

This compendium was prepared first in 2000 for the course *1E1142 Advanced Geodesy* (*Högre geodesi*), given to the fouth-year students of the surveying engineering programme at the Royal Institute of Technology (KTH). After revision in 2004, the compendium is used as the main literature for courses *Reference Systems* and *Physical Geodesy*, offered to students in the International Master Programme in Geodesy and Geoinformatics (IMPGG).

The compendium aims to introduce students to theories and methods for establishing large-scale geodetic networks and for studies of the figure and gravity field of the earth including its dynamic changes. There are two main subjects involved here which are both parallel and cross-linked to each other. The first subject is the technical aspect of geodesy as an applied technology, namely precise positioning on large scale using astrogeodetic triangulation, gravimetric methods and space techniques. The second subject is the scientific aspect of geodesy, i.e. the studies of the size, shape, gravity field and dynamic changes of the earth using observations of various kinds.

Rigorously speaking, the above two subjects are not separable. Defining and maintaining modern geodetic reference systems are closely related to both subjects. Therefore, this compendium will cover a variety of subfileds within geodesy, including geodetic astronomy, ellipsoidal geodesy, astrogeodetic triangulation, potential theory, physical geodesy, satellite geodesy and geodynamics. it is the author's hope that these subfields will not be felt by the course participants as separated and isolated elements, rather as integrated parts in a unified and systematic treatment of the main subjects.

It is assumed that students have already aqcuired background knowledge in surveying engineering before they take the above-mentioned courses. Therefore, the compendium will not treat practical field measurements. Instead, the emphasis is put on the theoretical concepts related to the scientific and technical problems of geodesy.

A list of related literatures, though far from complete, can be found at the end of the compendium. Many references are pages on the World Wide Web. In these cases, the date of the author's last visit to the pages concerned is given, not the date of creation or last revision of the concerned pages.

The author would like to thank Dr *Jonas Ågren*, Dr *Yuriy Reshetyuk* and students at KTH for proofreading and valuable comments.

($JD = 2\ 454\ 34.0.209$, $\phi = 59^0\ 21'\ 0.0''$, $\lambda = 18^0\ 04'\ 9.3''$) *Huaan Fan*

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Geodesy should better be called *geometry*, as the latter means earth (*geo-*) measurement (*-metry*). Literally, *geodesy* stands *for* earth (*geo-*) dividing (*-desy*). For ancient Egyptians who needed to measure and then divide land in the frequently flooded Nile region, land survey involved not only the theoretical principles of geometry but more importantly the sophisticated skills to apply the principles in practice. This might explain why the ancient Greek philosopher *Plato* once said that geodesy is an *art* while geometry is just a *technique.* The extension of land measurement to determine the global figure of the earth and fascinating astronomical observations together created the first ″world picture″ of the earth planet at the very beginning of our civilization.

Modern geodesy concerns studies of the figure and gravity field of the earth as well as the associated time-variable changes. It is not only a natural science on the figure and gravity field of the earth, but also a useful applied technology for positioning on and near the earth surface. One of the most important tasks of modern geodesy is to define and maintain geodetic reference systems on national or global scales to support production, utilization and distribution of geographic information to a broad user communities.

1.1 History of Geodesy

Ancient Geodesy

Ancient Hindus believed that the earth was just a big flat pancake supported by a number of pillars. Another earth model by Indians regarded the earth as a vast shell carried by four powerful elephants standing on the back of a giant turtle swimming in the world ocean (Bjerhammar,1962). About 600 B.C., *Pythagoras* realized that the earth is spherical. However, *Aristotle* was probably the first one to give proof for that. His first proof is that the shadow of the earth during a lunar eclipse is always circular. If the earth was not round, its shadow would not be circular at all directions. The second argument of Aristotle for a spherical earth is that stars rise earlier for people in the East than in the West, which could not happen if the earth was not spherical. He also noted that some stars could be seen in the North but could not be seen in the South, which implies that the earth is curved (Lightman, 1982).

The first field measurement for determining the size of a spherical earth was made by *Eratosthenes* (276-195 B.C.). He noted that on the first day of the summer, the sunlight struck the bottom of a vertical well in Seyne of Egypt, indicating the sun was exactly overhead, while the sun made an angle ($\Delta\phi$) with the vertical equal to 1/50 of a circle in Alexandria at the same time. These two places are roughly along the South-North direction. As the Sun is so far away, the solar rays arriving at both places are parallel and thus the geocentric angle between the two places are also equal to $\Delta\phi = 2\pi/50$ radians. Based on the time needed to travel by camels between these two places, he further estimated the distance ΔS between them to about 5000 stadia (1 stadia $\approx 180\ m$) or about 900 km. Finally Eratosthenes could calculate the radius of the spherical earth as (Cf **Figure 1.1**) :

$$R = \frac{\Delta S}{\Delta \phi} \approx 7200\ km \tag{1.1}$$

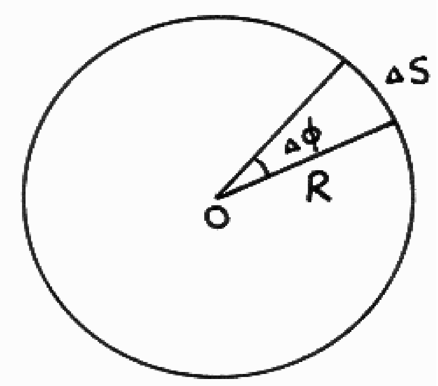

Figure 1.1: Determination of a spherical earth

Eratosthenes' result differ from modern results by about 13%. It is surprisingly good considering the primitive measurement techniques available at that time. This determination has also established the method known as *arc measurement along the meridian*, where the distance ΔS between two points at the same meridian and the corresponding geocentric angle $\Delta\phi$ are measured to estimate the earth radius. More than one thousand years later, new determinations based on this method were made in China (725 AD) and also in Arabia (827 AD).

French Expeditions

The accuracy of arc measurement along the meridian is limited, partially due to difficulty in distance measurements over long distances. In the earlier of 1600's, telescope was invented which led *Snellius* to develop the method of geodetic triangulation that greatly improved distance measurements. Around 1670, Frenchman *Picard* made triangulation near Paris and obtained the earth radius as $R = 6372\ km$. About five years earlier, *Isac Newton* already came to the idea on the gravitational law. However, the inaccurate earth radius at that time prevented him from verifying that the *ratio between the gravitational forces on the surfaces of the Moon and on the earth is inversely proportional to the ratio between their radius squared.* In 1682 when Newton learned Picard's result, he could eventually verify and publish the universal gravitational law.

The most significant event in the 18th century related to the determination of the figure of the earth is the controversy on the flattening of the earth. *Newton* (and also *Huygens*) believed that the earth is flattened toward the Poles, because the earth is rotating around the polar axis and it was once fluidlike. One consequence of such a flattening is that the meridian arc at higher latitude is longer than that at lower latitude for equal angular length. Around the turn of 1700's, French astronomer *Cassini* continued Picard's triangulation near Paris. In contrary to Newton's claim, his triangulation showed that the length of the meridian arc per degree of latitude difference in southern France was longer than that in the North.

To settle down this dispute on whether the earth is flattened toward the Poles or toward the equator, the French Academy sent two expedition teams to carry out arc measurements, one in South America near the equator and another in Lappland between Sweden and Finland. The measurement results confirmed Newton's claim and an ellipsoidal earth flattened toward the Poles was finally established in the scientific community.

The results of the arc measurements along the meridian could be used to determine the two ellipsoidal parameters of the earth ellipsoid (equatorial radius a and first eccentricity squared e^2). One important result was Bessel's earth ellipsoid, once widely used in Europe and still used in Sweden. Arc measurements were also made along the parallel circles, but with less success due to inaccurate time (longitude) determination.

Astrogeodetic Triangulation

An obvious disadvantage of arc measurements was that they determine only the size and shape of the earth along several measured arcs, neglecting most parts of the earth surface. An improvement is to run large scale *astrogeodetic triangulation* across the country or continents, in which a small number of absolute positions, azimuths and baselines and a large number of triangular angles (or directions) are measured.

Astrogeodetic triangulation could be quickly spread all over the world, not because of its potential for determining the figure of the earth, rather because of its economical and political importance for general mapping, military operations, urban and rural development, engineering projects, etc. However, the determination of the figure of the earth and the practical implementation of large scale triangulation are closely interrelated to each other, since geodetic observations made on the irregular surface of the earth must be reduced to and finally computed on some mathematically suitable surface. Here, the use of an *ellipsoid of revolution* as a mathematical representation of the earth's figure turned out to be a proper solution for computations of astrogeodetic triangulation networks. Obviously, to determine the earth ellipsoid from triangulation results, one need to go through an "iterative" process.

Within the context of large scale astrogeodetic triangulation across the whole Europe during the 19th century, one probably should mention the triangulation work near Hannover during the period 1818-1825, which led *Carl Friedrich Gauss* to invent many fundamental concepts of *differential geometry.* For example, geodetic measurement of distances on the surface of an ellipsoidal earth gave birth to the mathematical definition of the shortest curve between two points on an curved surface, commonly known as the *geodesic,* or *geodetic line.* The least squares method and the normal distribution of errors are other important contributions of *Gauss* to mathematics, which were associated with his geodetic and astronomical activities.

Historically, the scale of a astrogeodetic triangulation network is defined by a fewer baselines, normally measured with invar tapes and likely extended. The invention of *electromagnetic distance measurement* (EDM) instruments, Geodimeter, by the Swedish geodesist *Erik Bergstrand* opened a new era in the history of geodesy. With geodimeter, large number of sides in a triangulation network can be measured, directly, easily and accurately.

Modern Geodesy

During the development of astrogeodetic method, it became more and more clear that determination of the earth's figure with geodetic measurements is not a pure geometric problem rather also involving the gravity field of the earth. The combination of astronomical and geodetic observations required knowledge on the angle between the gravity vector (the plumb line) and the ellipsoidal normal, known as *deflection of the vertical.* The use of a regional reference earth ellipsoid for a country or continent also raised the question on the position of the ellipsoidal centre with respect to the gravity centre of the earth. The position of a reference ellipsoid assumes that the geoid height(s) at the geodetic initial point(s) are known.

As triangulation networks are basically two dimensional, topographical heights are determined separately using levelling techniques with respect to the geoid (roughly the mean sea level), which is nothing else but an equipotential surface of the earth's gravity field. Due to the non-parallelism of level surfaces of the earth's gravity field, geometrical levelling does not provide unique heights over the geoid and thus gravity measurements are needed to obtain the geopotential of benchmarks. Now, geodesy has evolved from primarily studying the geometric figure of the earth to also including the determination of the earth's gravity field.

Meanwhile, the determination of the earth's figure began to shift from directly determining the earth surface to studying the geoid. This is partially due to the fact that the earth's surface is very rough (as high as nearly 9 km in Mt.Everest and as low as 10 km in deep oceans), while the geoid is much smoother (±110 metres with respect to an ellipsoid of revolution). The more important reason was probably that the geoid is so closely associated with the earth's gravity field that the use of the geoid as an approximation of the earth's physical surface will be of great benefit to studies of the earth's physical properties.

As approximately two thirds of the earth surface is covered by oceans and many continental areas are still unsurveyed, geodetic measurements (of either geometrical or physical characters) made on the earth surface are essentially limited for fully determining the figure and the gravity field of the earth. A breakthrough came when man-made artificial satellites and other space technologies found applications in geodesy. This breakthrough has revolutionized geodesy, as artificial satellites are especially suitable for observing the global features of the earth, including the geometrical figure and the gravitational field. Since the launch of the first artificial satellite, various satellite techniques have been utilized for geodetic purposes. The latest and also the most revolutionizing satellite technology used in geodesy is probably the *Global Positioning System* (GPS).

As measurement techniques got improved, geodesists began to get aware of dynamic changes going on within and around the earth. At one hand, these changes (e.g. the change of the earth rotation axis, regional and global crustal movement, etc.) will cause most geodetic observations and eventually the earth's figure and gravity field to change. On the other hand, geodetic theory and especially geodetic measurement techniques have turned out to be very useful in monitoring and consequently helping explain many dynamic processes and phenomena. As interdisciplinary sciences develop, geodynamics becomes an important part of the geodetic science. This new development also helps place geodesy within the domain of natural sciences and particularly geosciences, in contrary to the applied characters of practical geodetic measurement techniques.

In summary, geodesy is one of the oldest natural sciences in the human history. Scientifically, geodesy deals with the *study of the figure of the earth, its gravity field and their dynamic changes*. These study tasks are closely interrelated with each other. For practical convenience, one may divide geodesy into three subfields: *geodetic positioning*, *gravity field study* and *geodynamics*.

From technical viewpoints, geodetic technology consists of both conventional terrestrial techniques (triangulation, levelling, etc.) and modern space techniques. Geodetic techniques are used not only for scientific purposes, but also for establishing national and global geographical infrastructure (i.e. national/continental geodetic control networks), for general mapping, for engineering construction and for urban and rural planning. As geodesy develops and its applications get wider and wider, new subdisciplines emerge out of geodetic science, such as *photogrammetry* originally for topographical mapping by measuring aerial photographs, *cartography* for graphical presentation of land survey results, and recently *geoinformatics* for integrated handling of geospatial information using modern computer technology. In a wide sense, the subject of geodesy should include all of the subjectes mentioned above. For this reason, we also use the name *Surveying engineering* or *Geomatics engineering.*

Although geodesy is an old science, it is still deveoping and contributing to increased, improved and new knowledge on our planet. In the new millenium geodesy faces new challenges such as:

- defining and maintaining modern, accurate, real-time, geodetic reference systems with the help of advancements in computer engineering, space techniques and telecommunication to facilitate production, distribution and use of geographic information of various kinds
- improvement in the determination of the earth's gravity, e.g. with the help of new dedicated satellite gravity missions (CHAMP, GRACE, GOCE, etc)
- measuring and understanding geodynamic processes using geodetic measurements techniques

The Role of Geodesy in the Era of GIS

During the last decades, Geographical Information Systems (GIS) have found wide applications in many fields involving geographically related information such as, just to mention several examples, urban planning, environmental monitoring, land and real estate management, natural resource management, facility management, and so on. Although most geospatial information in GIS databases is collected using "traditional" surveying technologies (geodetic, photogrammetric or remote sensing), GIS have opened new, unpreceedented application areas for geospatial information.

Spatial coordinates are the most important and most fundamental component in all GIS databases. Coordinates should be accurate, reliable, easily accessible and most of all, refer to certain mathematically and physically well-defined coordinate system. One of geodesy's most important tasks is to define and maintain geodetic reference systems for all geospatial information. A significant part of today's GIS databases use horizontal coordinates referred to the old astrogeodetic triangulation networks and vertical coordinates referred to the classical levelling networks. In the near future, all new geospatial information will be produced in well-defined three-dimensional (3D) coordinate systems. Therefore there is a need for both geodetic engineers and GIS users to understand:

- *how traditional astro-geodetic triangulation networks and levelling networks are built;*
- *how modern 3D reference systems are defined;*
- *relationships between the traditional networks and modern networks and*
- *transformation between different types of coordinate systems.*

In the era of Geographical Information Systems (GIS), geodetic surveying method will continue to be one of the major technologies for geospatial information acquisition. In addition to traditional techniques using theodolites and totalstations, new geodetic technology such as laser scanning and integrated surveying technology. One example of the latter is the so called *Mobile Mapping System* where GPS, INS (Inertial Navigation System) and digital cameras are integrated in one system for fast, accurate and real-time positioning.