Modeling tropospheric delays in space geodetic techniques

Daniel Landskron

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1. Fundamentals
2. Modeling delays in the troposphere
3. Vienna troposphere models
4. Conclusion
5. Outlook
I will shortly introduce some physics behind tropospheric delays and then talk about fundamentals of troposphere modeling, sorry for those who already know this.
1. Fundamentals

Troposphere

- Troposphere delays: strictly speaking delays in the neutral atmosphere (up to 100 km)
- Radio signals are delayed and bent due to interaction with gases and water particles => refractivity
- Essentially no frequency dependence across microwave regime
- Small frequency dependence for optical techniques (Satellite Laser Ranging)
D1 unlike the ionosphere, which is indeed frequency dependent and whose effect can therefore be eliminated through measuring in two frequency bands
D. Landskron; 2017-10-03

D8 the neutral atmosphere is defined as that part of the atmosphere which is non-ionized, contrary to the ionosphere whereas the troposphere is that part of the atmosphere below the first temperature inversion.

However, spatially those two coincide, why the sloppy term “troposphere” is used
D. Landskron; 2017-10-03
Refractivity

- Strictly speaking, refractivity is a complex number
  \[ N = N_0 + N'(\nu) - i N''(\nu) \]

- Real part: causes refraction and propagation delays
- Imaginary part: causes absorption; important for water vapour radiometers

Refractivity of microwaves

Figure 21: The total refractivity as function of frequency. The total pressure is 1013 hPa, the temperature 300 K, and the relative humidity is 100 %. Three different cases are shown corresponding to different concentrations of liquid water: 0 g/m³, 0.05 g/m³, and 1 g/m³.
as water vapour radiometers are capable of measuring the delay of a signal arising from water vapor
D.Landskron; 2017-10-03

but in the following we only consider the real part, that is, we neglect the attenuation of the signal
D.Landskron; 2017-10-03

this term here (ν) is frequency
Daniel; 2017-10-12

as the frequencies of GNSS lie slightly above 1 GHz and VLBI below 10 GHz as well, we can consider
the refractivity to be frequency independent (visible through the straight line)

in the optical range, this is different
D.Landskron; 2017-10-03
Refractivity of microwaves

Distinguished between a hydrostatic part and a wet part

\[ N = k_1 \frac{R}{M_d} \rho + k_2' \frac{P_w}{T} T_\text{w}^{-1} + k_3 \frac{P_w}{T^2} T_\text{w}^{-1} = N_h + N_w \]

\[ k_2' = k_2 - k_1 \frac{M_w}{M_d} \]

Wet part: surface values not representative for the upper air conditions

Figure 22: Examples of vertical profiles of the hydrostatic and wet refractivity. The profiles are calculated using radiosonde data from Vienna, Austria.
there would be a further term for liquid water, but it is not contained in this formula

Daniel; 2017-10-12
Optical refractivity of moist air

- $k_3$ can be ignored; Wet part smaller
- Small frequency-dependency

Figure 23: The total optical refractivity as function of frequency. The total pressure is 1013 hPa, the Temperature 300 K, and the relative humidity is 100%.

2. Modeling delays in the troposphere
D11 say: the wet part is approximately 70 times smaller
D.Landskron; 2017-10-03
Definition of path delay in the neutral atmosphere

Figure 24: Path taken by a signal through the atmosphere. The signal will take the path with the shortest propagation time (S). Since the signal propagates slower in the atmosphere than in vacuum, the geometrical length of S will be larger than the straight path G.

Bending effect [S - G] about 0.2 m at 5° elevation (added to the hydrostatic mapping function)

Delays in zenith direction

- Zenith hydrostatic delay
  - Ca. 2.3 m at sea level
  - Can be determined very accurately from \( p \) (mm-accuracy) + Saastamoinen (1972)

- Zenith wet delay
  - Ca. 0.05 - 0.4 m at sea level
  - Rule of thumb: \( \Delta L_w [\text{cm}] \approx P_{\text{ref}} [\text{hPa}] \)
  - Can only be approximated from surface data
    GPT2/GPT3 + Askne & Nordius (1987)
Bild 11

D12  this is a simplified graph how the bending looks like  
     D.Landskron; 2017-10-03

D3   Das Beispiel mit dem Lifesaver bringen  
     Daniel; 2017-10-14

Bild 12

D13  and decreases with increasing height  
     D.Landskron; 2017-10-03

D14  approximate station position is also needed  
     D.Landskron; 2017-10-03

D16  because the wet refractivity is highly variable in the vertical column  
     D.Landskron; 2017-10-03
Pressure values

- Simple empirical models like Berg (1948) and Hopfield (1969)

\[
p = 1013.25 \cdot (1 - 0.0000226h)^{5.225}
\]

\[
p = 1013.25 \cdot \left( \frac{T_k - \alpha h}{T_k} \right) \frac{r}{a}^{0.28}
\]

- More sophisticated models like
  - UNB3m (5 latitude bands, annual with fixed phase)
  - GPT (9x9 spherical harmonics, annual with fixed phase)
  - GPT2/GPT3 (5°x5° or 1°x1° grid, annual + semi-annual terms)

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Figure 25: Pressure values for station O’Higgins in Antarctica from the ECMWF (grey line), local pressure recordings at the radio telescope (red squares), GPT (blue line), and pressure determined with the model by Berg (1948) (black line)
Precipitable water

- Integrated water vapour IWV in kg/m$^2$
  \[ IWV = \frac{10^6 ZWD}{k'_2 + \frac{k_3}{T_m} R_v} \]

- Precipitable water PW in m
  \[ PW = \frac{IWV}{\rho_t} \]

- PW is approximately 1/6 of the zenith wet delay

Water vapor comparison
Modeling troposphere delays

Assuming Azimuthal Symmetry:

\[ \Delta L(e) = \Delta L^z_h \cdot mf_h(e) + \Delta L^z_w \cdot mf_w(e) \]

- \( \Delta L(e) \): total delay dependent on elevation
- \( \Delta L^z_h \): hydrostatic delay in zenith direction; can be modeled a priori
- \( \Delta L^z_w \): wet delay in zenith direction; approximated or estimated in data analysis
- \( mf(e) \): mapping function (\( mf_h > mf_w \))

Mapping functions

- Mapping function not perfectly known
- Errors via correlations also in station heights (and clocks)
- Low elevations necessary to de-correlate heights, clocks, and zenith delays
- Rule of thumb: the station height error is about 1/5 of the delay error at 5° elevation (if cutoff angle is 5°)
Mapping functions

- Continued fraction form (Herring, 1992)

\[
mf(e) = \frac{1 + \frac{a}{b}}{1 + \frac{1 + c}{a}} \cdot \frac{\sin(e) + \frac{b}{\sin(e) + e}}{\sin(e) + \frac{b}{\sin(e) + e}}
\]

Mapping function models

- Saastamoinen (1972), Chao (1974), CfA2.2 (Davis et al., 1985), ...
- MTT: MIT Temperature mapping functions (Herring, 1992)
- NWF: New Mapping Functions (Niell, 1996)
- IMF: Isobaric Mapping Functions (Niell, 2000)
- VMF: Vienna Mapping Functions (Böhm et al., 2006)
- GMF: Global Mapping Functions (Böhm et al., 2006)
- GPT2/GPT2w (Lagler et al., 2013, Böhm et al., 2015)
- VMF3/GPT3 (Landskron and Böhm, 2017)
Modeling troposphere delays

Assuming Azimuthal Asymmetry:

\[ \Delta L(a, e) = \Delta L_n^2 \cdot mf_h(e) + \Delta L_w^2 \cdot mf_w(e) + mf_g(e) \cdot (G_n \cos a + G_e \sin a) \]

- \( \Delta L(a, e) \): total delay dependent on azimuth and elevation (m)
- \( \Delta L^2 \): delay in zenith direction (m)
- \( mf(e) \): mapping function
- \( G_n \): north gradient (m)
- \( G_e \): east gradient (m)

Horizontal gradients

- Horizontal gradients due to:
  - Atmospheric bulge
  - Weather fronts
  - Coastal conditions

- Chen and Herring (1997)

\[ \Delta L(a, e) = \Delta L_0(e) + mf_g(e)(G_n \cos a + G_e \sin a) \]

\[ mf_g(e) = \frac{1}{\sin(e) \tan(e) + C} \]

\[ C_n = 0.0031, \ C_w = 0.0007 \]

- Typical gradient: 1 mm (corresponds to 0.1 m delay at 5° elevation)
Horizontal gradients

• Correspond to tilting of the mapping function

![Diagram](image)

Figure 33: Tilting of the mapping function by the angle $\beta$ assuming a horizontally stratified atmosphere

Horizontal gradient models

Gradients are either estimated in the analysis or they are determined from external data (e.g. NWM)

A priori models:

• **DAO** (MacMillan and Ma, 1997)
• **LHG** (Böhm and Schuh, 2007)
• **APG** (Böhm et al., 2013)
• **GRAD** (Landskron et al., 2016)
• **GPT3** (Landskron et al., 2017)
Ray-tracing

- To find the ray-path from the source to the telescope (iterative calculation)
- Coupled differential equations need to be solved
- 1D, 2D or 3D ray-tracing
- Feasible for VLBI but probably not for GNSS
- Basis for most accurate mapping functions and gradient models (VMF series)

Figure 27. Geometry of a 1D ray-tracing method, for a receiver located at $P_1$ and the upper limit of the troposphere at $P_2$. Points $P_3$ and $P_4$ show two sample points of the ray path. The $y$ and $z$ axis of the Cartesian coordinate system are parallel to horizon and zenith direction at the site, respectively. $S_2 = |P_3 - P_4|$ is the distance between two successive points along the path.
D18  explain, what would be different in 2D and 3D ray-tracing
D.Landskron; 2017-10-03
Water vapour radiometry

- WVR estimate the wet delay by measuring the thermal radiation from the sky
- At microwave frequencies where the atmospheric attenuation due to water vapour is rather high
- WVR do not work during rain or below 15° elevation

Atmospheric delays for SLR

- Wet part much smaller than for microwaves
- Only modeled, not estimated
- Thus, better estimation of height compared to horizontal components
- Theoretical possibility to estimate troposphere delay with two frequencies, but accuracy of delays not yet sufficient for that
just like it is done with the ionosphere
D.Landskron; 2017-10-05
Textbook

- Very detailed description of tropospheric delays

3. Vienna Troposphere Models
it also describes the physical background of path delays very accurately, which I spared in this presentation for the most part. 

but, only models before 2013 are considered, that is, no GPT2, GPT3, VMF3, GRAD.
Vienna models

- TU Wien has become main provider of troposphere models
- Applicable for GNSS and VLBI analysis
- Included in important software as well as realizations (Bernese, ITRF,..)

VLBI

- Plane wavefronts because of huge distance (~10 billion ly)
- Determine phase difference \( \tau \) between 2 sites
- Correct for errors (ionosphere, troposphere,..)

- Station positions and velocities, source positions, zenith wet delay
I think I do not have to explain GNSS or GPS, but here is a short introduction to VLBI, as I think that not everybody knows about it.

D. Landskron, 2017-10-04
Mapping functions

- **Discrete mapping functions**
  - VMF: Vienna Mapping Functions (Böhm and Schuh, 2004)
  - VMF1: Vienna Mapping Functions 1 (Böhm et al., 2006)
  - VMF3: Vienna Mapping Functions 3 (Landskron and Böhm, 2017)

- **Empirical mapping functions**
  - GMF: Global Mapping Functions (Böhm et al., 2006)
  - GPT: Global Pressure and Temperature (Böhm et al., 2007)
  - GPT2w: Global Pressure and Temperature 2 (Lagler et al., 2013)
  - GPT2w: Global Pressure and Temperature 2 wet (Böhm et al., 2015)
  - GPT3: Global Pressure and Temperature 3 (Landskron and Böhm, 2017)

- **Hybrid Model**
  - SA-GPT2w: Site-Augmented GPT2w (Landskron et al., 2015)

http://ggosatm.hg.tuwien.ac.at

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Figure 30: Hydrostatic mapping functions VMF1 and GMF at 5° elevation at Fortaleza, Brazil. Phenomena such as the El Niño event in 2009 cannot be captured with empirical mapping functions like GMF that contain only average seasonal terms.
Zunächst den Unterschied zwischen discrete und empirical erklären, und dann zu allen Modellen ein paar Wörter sagen
D.Landskron; 2017-10-03

Site-augmentation using in situ meteorological data
D.Landskron; 2017-10-03
Vienna Mapping Functions

- Determined from ray-traced delays through NWM from ECMWF
- Empirical functions for $b$ and $c$ coefficients
- All information from ray-tracing is condensed into the $a$ coefficients
- Available 6-hourly, either at VLBI/GNSS stations or on a global grid

\[
m(e) = \frac{1 + \frac{a}{b} \left(1 + \frac{1 + c}{1 + \frac{a}{\sin(e)}}\right)}{\sin(e) + \frac{b}{\sin(e) + c}}
\]

variable in time and space

ray-tracing

analytical functions
the grid is particularly important for GNSS users as they thus can produce zenith delays + mapping functions for any point on Earth.

D.Landskron, 2017-10-05
**VMF1 vs. VMF3**

<table>
<thead>
<tr>
<th>VMF1</th>
<th>VMF3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>b, c</strong></td>
<td><strong>b, c</strong></td>
</tr>
<tr>
<td>from 3 years of data on a 10°x10° grid</td>
<td>from 10 years of data on a 2.5°x2.0° grid</td>
</tr>
<tr>
<td>lat. dep. for <strong>c</strong>&lt;sub&gt;h&lt;/sub&gt;</td>
<td>lat. and lon. dep. for <strong>b</strong>&lt;sub&gt;h&lt;/sub&gt;, <strong>b</strong>&lt;sub&gt;w&lt;/sub&gt;, <strong>c</strong>&lt;sub&gt;h&lt;/sub&gt; and <strong>c</strong>&lt;sub&gt;w&lt;/sub&gt; through spherical harmonics (n=m=12)</td>
</tr>
<tr>
<td>annual variation for <strong>c</strong>&lt;sub&gt;h&lt;/sub&gt;</td>
<td>annual and semi-annual terms for <strong>b</strong>&lt;sub&gt;h&lt;/sub&gt;, <strong>b</strong>&lt;sub&gt;w&lt;/sub&gt;, <strong>c</strong>&lt;sub&gt;h&lt;/sub&gt; and <strong>c</strong>&lt;sub&gt;w&lt;/sub&gt;</td>
</tr>
<tr>
<td><strong>a</strong></td>
<td><strong>a</strong></td>
</tr>
<tr>
<td>strictly for el = 3.3°</td>
<td>LSM for el = [3°, 5°, 7°, 10°, 15°, 30°, 70°]</td>
</tr>
<tr>
<td>simple 1D ray-tracer</td>
<td>2D ray-tracer “RADIATE” (Hofmeister, 2016)</td>
</tr>
</tbody>
</table>

**Vienna Mapping Functions 3**

Spherical harmonics expansion for coefficients **b** and **c** up to degree and order 12
and on their basis, new a coefficients were calculated from the ray-traced delays
D.Landskron; 2017-10-05
Global Mapping Functions (GMF)

- GMF: “Averaged” VMF
- Spherical Harmonics up to degree and order 9 for \(a\), \(b\) and \(c\) from VMF1
- Annual variation with fixed phase (January 28)
  \[a = a_0 + A \cdot \cos \left( \frac{\text{doy} - 28}{365.25} \cdot 2\pi \right),\]
  \[a_0 = \sum_{n=0}^{9} \sum_{m=0}^{n} P_{nm}(\sin \theta)(A_{nm} \cos(m\lambda) + B_{nm} \sin(m\lambda))\]

Global Pressure and Temperature 2 (GPT2)

- Refined combination of GMF and GPT + additional parameters
- Not based on spherical harmonics, but on a grid-wise representation
- Bilinear interpolation from grid to desired location

\[b(t) = A_0 + A_1 \cos \left( \frac{mjd}{365.25} \cdot 2\pi \right) + B_1 \sin \left( \frac{mjd}{365.25} \cdot 2\pi \right)\]
\[+ A_2 \cos \left( \frac{mjd}{365.25} \cdot 4\pi \right) + B_2 \sin \left( \frac{mjd}{365.25} \cdot 4\pi \right)\]
\[+ k \cdot mjd\]
GPT2w is actually only a refinement of GPT2 regarding the wet quantities
D. Landskron; 2017-10-03
### GPT2w vs. GPT3

<table>
<thead>
<tr>
<th>GPT2w</th>
<th>GPT3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b, c$</td>
<td>$b, c$</td>
</tr>
<tr>
<td>from VMF1</td>
<td>from VMF3</td>
</tr>
<tr>
<td>$a$</td>
<td>$a$</td>
</tr>
<tr>
<td>1°x1° or 5°x5° grid</td>
<td>1°x1° or 5°x5° grid</td>
</tr>
<tr>
<td>annual and semi-annual terms</td>
<td>annual and semi-annual terms</td>
</tr>
<tr>
<td>$mf$ height correction by Niell (1996) for hydr. part</td>
<td>new $mf$ height correction for hydr. and wet part</td>
</tr>
<tr>
<td>-</td>
<td>horizontal gradients grid</td>
</tr>
<tr>
<td>1D ray-tracer</td>
<td>2D ray-tracer “RADIATE” (Hofmeister, 2016)</td>
</tr>
<tr>
<td>ECMWF monthly means 2001-2010</td>
<td>ECMWF monthly means 2001-2010</td>
</tr>
</tbody>
</table>

### Global Pressure and Temperature 3

Data fitting in order to derive empirical information

$a_h$: mean value and annual variation
**Input/output quantities**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>mjd</td>
<td>Modified Julian date</td>
<td></td>
</tr>
<tr>
<td>ω</td>
<td>Geographic latitude</td>
<td>rad</td>
</tr>
<tr>
<td>λ</td>
<td>Geographic longitude</td>
<td>rad</td>
</tr>
<tr>
<td>zd</td>
<td>Zenith-distance (θ−elevation)</td>
<td>rad</td>
</tr>
<tr>
<td>mh</td>
<td>Hydrostatic mapping factor</td>
<td></td>
</tr>
<tr>
<td>mw</td>
<td>Wet mapping factor</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>Pressure</td>
<td>hPa</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
<td>°C</td>
</tr>
<tr>
<td>dT</td>
<td>Temperature lapse</td>
<td>K km⁻¹</td>
</tr>
<tr>
<td>Tₚ</td>
<td>Mean temperature weighted with</td>
<td></td>
</tr>
<tr>
<td></td>
<td>water vapor pressure</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>Water vapor pressure</td>
<td>hPa</td>
</tr>
<tr>
<td>aₜ</td>
<td>Hydrostatic mapping function</td>
<td></td>
</tr>
<tr>
<td></td>
<td>coefficient (valid at sea level)</td>
<td></td>
</tr>
<tr>
<td>a₁</td>
<td>Wet mapping function coefficient</td>
<td></td>
</tr>
<tr>
<td>λₜ</td>
<td>Water vapor depression factor</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>Good indication</td>
<td>m</td>
</tr>
<tr>
<td>Gₙh</td>
<td>Hydrostatic north gradient</td>
<td>m</td>
</tr>
<tr>
<td>Gₚh</td>
<td>Hydrostatic west gradient</td>
<td>m</td>
</tr>
<tr>
<td>Gₙw</td>
<td>Wet north gradient</td>
<td>m</td>
</tr>
<tr>
<td>Gₚw</td>
<td>Wet east gradient</td>
<td>m</td>
</tr>
</tbody>
</table>

**Handling for user**
so we see that GPT3 acts as a complete troposphere model which outputs all information that may be required in troposphere modeling

D.Landskron, 2017-10-03
Mapping functions comparison

![Mapping functions comparison](image)

Mapping functions comparison

![Mapping functions comparison](image)
Delay comparison

Differences in slant total delay to ray-tracing (mm)
2592 grid points
120 epochs (2001-2010)
el = 5°

VMF1

VMF3

Delay comparison

Differences in slant total delay to ray-tracing (mm)
2592 grid points
120 epochs (2001-2010)
el = 5°

GPT2w

GPT3
Mean absolute error (MAE) in slant delay w.r.t. ray-tracing (mm)
2592 grid points
120 epochs (2001-2010)
\( \epsilon' = 5^\circ \)

<table>
<thead>
<tr>
<th>(mm)</th>
<th>( \Delta l )</th>
<th>( \Delta l_h )</th>
<th>( \Delta l_w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>VMF1</td>
<td>1.73</td>
<td>1.67</td>
<td>0.30</td>
</tr>
<tr>
<td>VMF3</td>
<td>0.82</td>
<td>0.73</td>
<td>0.30</td>
</tr>
<tr>
<td>GPT2w</td>
<td>6.85</td>
<td>6.10</td>
<td>1.63</td>
</tr>
<tr>
<td>GPT3</td>
<td>6.46</td>
<td>5.68</td>
<td>1.60</td>
</tr>
</tbody>
</table>

Mean absolute diff in slant total delay w.r.t. ray-tracing (mm)
33 sites around the world
1999-2014

<table>
<thead>
<tr>
<th>[mm]</th>
<th>3°</th>
<th>5°</th>
<th>7°</th>
<th>10°</th>
</tr>
</thead>
<tbody>
<tr>
<td>VMF1</td>
<td>0.52</td>
<td>3.98</td>
<td>2.54</td>
<td>1.47</td>
</tr>
<tr>
<td>VMF3</td>
<td>1.17</td>
<td>2.64</td>
<td>1.66</td>
<td>0.91</td>
</tr>
<tr>
<td>GPT2w (1°x1°)</td>
<td>54.13</td>
<td>18.95</td>
<td>8.35</td>
<td>3.27</td>
</tr>
<tr>
<td>GPT3 (1°x1°)</td>
<td>53.68</td>
<td>18.90</td>
<td>8.30</td>
<td>3.24</td>
</tr>
</tbody>
</table>
there is a lot of information in this plot: auf alles eingehen
D.Landskron; 2017-10-04
Delay comparison

Mean difference w.r.t. ray-tracing (mm)
33 sites around the world, 1999-2014, el = 5°

BLR comparison

- Baseline Length Repeatability (BLR) from VLBI analysis, good tool for assessing accuracy of geodetic products
- Analysis with Vienna VLBI and Satellite Software (VieVS)
- Hardly any difference between the mapping function models
- Main influence from zenith delays, mapping functions not that effective
- Estimation of zenith wet delays very accurate
D33 if somebody is interested in learning how to use VieVS as a VLBI analysis software
D.Landskron; 2017-10-04

D34 not even between empirical and discrete models
D.Landskron; 2017-10-04
1. Empirical $\Delta L_w$ from GPT2w
2. Measure $T$ and $e$ in situ
3. Augment the empirical $\Delta L_w$

$$zwd = zwd_{GPT2w} + M_1 \times (T_{in\,situ} - T_{GPT2w}) + M_2 \times (e_{in\,situ} - e_{GPT2w})$$

Universal, global coefficients $M_1, M_2$:
- $M_1 = 4.8 \times 10^{-4}$ [m/°C]
- $M_2 = 0.00915$ [m/hPa]
Comparison of $\Delta L_z$ for BZRG

Site-augmented GPT2w
when measuring $T$, we get the green line which is already slightly closer to the real data
D. Landskron; 2015-10-14
Comparison of $\Delta L_z$ for BZRG

Comparison of $\Delta L_z$ for ALIC
when also measuring water vapor pressure, then the maximum improvement is achieved
D.Landskron; 2015-10-14
Comparison of $\Delta L^w_z$ for ALIC

Site-augmented GPT2w

IGS
GPT2w
GPT2w + T
GPT2w + T, e

Comparison of $\Delta L^w_z$ for ALIC

Site-augmented GPT2w

IGS
GPT2w
GPT2w + T
GPT2w + T, e
Comparison of $\Delta L_w$ for NYA1

Site-augmented GPT2w

- IGS
- GPT2w
- GPT2w + T
- GPT2w + T, e

Graph showing the comparison of zenith wet delay for NYA1 using different models.
Comparison of $\Delta L_z$ for NYA1

Site-augmented GPT2w

Mean absolute error (MAE) in zenith wet delay to ray-tracing (cm)
33 sites around the world
1999-2014

<table>
<thead>
<tr>
<th>(cm)</th>
<th>zwd</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPT2w</td>
<td>2.8</td>
</tr>
<tr>
<td>GPT2w+ T</td>
<td>2.7</td>
</tr>
<tr>
<td>GPT2w+ T and $e$</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Site-augmented GPT2w

- GPT2w well suited for site-augmented approach using in situ measurements of $T$ and $e$
- In situ measurement of $T$ yields small improvement in zenith wet delay $\Delta L_w^z$ (~5%)
- Additional in situ measurement of $e$ yields significant improvement in zenith wet delay $\Delta L_w^z$ (~30%)
- In general, best performance of site-augmented GPT2w is achieved in dry regions

Horizontal gradients

- Discrete a priori gradient models
  - LHG: Linear Horizontal Gradients (Böhm and Schuh, 2007)
  - GRAD (Landskron et al., 2016)

- Empirical a priori gradient models
  - APG: A priori gradients (Böhm et al., 2013)
  - GPT3: Global Pressure and Temperature 3 (Landskron and Böhm, 2017)

http://ggosatm.hg.tuwien.ac.at
the list of a priori gradient models is less comprehensive, because very often such models are not used in analysis at all, as the gradients are estimated in the data analysis though least-squares adjustment
A priori gradients GRAD

- Determined from 2D-raytracing at 7 elevations and 16 azimuths through LSM
- For all VLBI measurements
- 6-hourly (at each NWM epoch)

\[
\Delta L(a, e) = \Delta L_0(e) + m f_g (G_n \cos a + G_c \sin a) = \text{GRAD-1}
\]

\[
\Delta L(a, e) = \Delta L_0(e) + m f_g (G_n \cos a + G_c \sin a + G_{n2} \cos 2a + G_{c2} \sin 2a) = \text{GRAD-2}
\]

Residuals between ray-traced delays and modeled delays

No gradients

GRAD-1

GRAD-2

GRAD-3
the most precise a priori gradients available are the gradients GRAD. They are split into 3 versions, depending on three different gradient formulas, with GRAD-1 being the main model however.
A priori gradients GRAD

Higher-order gradients improve delays
WETZELL, September 2011

Higher-order gradients smaller in size
WETZELL, September 2011

Global Pressure and Temperature 3 (GPT3)

Fig. 3 Mean values $A_0$ (top left), annual amplitudes $A_1$ (top right), semi-annual amplitudes $A_2$ (bottom left) and standard deviation of the residuals (bottom right) of the hydrostatic north gradient $G_{an}$ from GPT3.
Gradient comparison

Empirical gradients only describe a fraction of the real gradients

WITTZELL, 06/2014–12/2014

- $G_1$ (mm) vs. mjd
- $G_2$ (mm) vs. mjd

D43
GRAD
DAO
GPT3
Gradient comparison

Mean absolute residuals (mm) between ray-tracing and VMF3 + gradient models at el = 5°

<table>
<thead>
<tr>
<th>Gradient Model</th>
<th>Mean Abs. Diff. in ΔL (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>α = 0°</td>
</tr>
<tr>
<td>no a priori gradients</td>
<td>25.6</td>
</tr>
<tr>
<td>GRAD-1</td>
<td>4.1</td>
</tr>
<tr>
<td>GRAD-2</td>
<td>1.4</td>
</tr>
<tr>
<td>GRAD-3</td>
<td>1.4</td>
</tr>
<tr>
<td>APG</td>
<td>16.4</td>
</tr>
<tr>
<td>GPT3</td>
<td>9.4</td>
</tr>
</tbody>
</table>

Baseline length repeatability (BLR) from 1338 VLBI sessions from 2006-2014

<table>
<thead>
<tr>
<th>(cm)</th>
<th>NO estimation</th>
<th>WITH estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ray-tracing</td>
<td>1.57</td>
<td>1.64</td>
</tr>
<tr>
<td>No a priori gradients</td>
<td>1.68</td>
<td>1.65</td>
</tr>
<tr>
<td>LHG</td>
<td>1.66</td>
<td>1.67</td>
</tr>
<tr>
<td>GRAD-1</td>
<td>1.58</td>
<td>1.66</td>
</tr>
<tr>
<td>GRAD-2</td>
<td>1.57</td>
<td>1.65</td>
</tr>
<tr>
<td>DAO</td>
<td>1.64</td>
<td>1.66</td>
</tr>
<tr>
<td>GPT3</td>
<td>1.63</td>
<td>1.66</td>
</tr>
</tbody>
</table>
Gradients results

- GRAD yield best performance of all a priori gradients
- Use of a priori gradients in VLBI analysis is very important
- Estimation only makes sense when enough observations
- Empirical gradients may be valuable for GNSS

4. Conclusions
Because they can be produced for any point on Earth
D. Landskron, 2017-10-04
Conclusions

- Many troposphere models from TU Wien
- Mapping functions + horizontal gradients
- Applicable for GNSS and VLBI analysis
- VMF1 and GMF most important ones
- Ray-tracing through NWM best approach

Conclusions

- Several new/refined models created
  - Mapping functions + horizontal gradients
  - All of which outperform predecessors, but only to a small degree
- Tropospheric modeling close to peak of technical means?
  - (Reference) ray-traced delays approximated very well
  - Denser and more accurate NWM
  - Improved strategies and concepts
yielding accuracies which by far surpassed those from before
D.Landskron, 2017-10-05
5. Outlook

• Operationally provide VMF3/GRAD
  – for all IVS stations (VLBI)
  – for all IGS stations (GNSS)
  – for all IDS stations (DORIS)
  – on a grid
  – for Satellite Laser Ranging (SLR)

• Distribute all data via:
  [http://ggosatm.hg.tuwien.ac.at](http://ggosatm.hg.tuwien.ac.at)
and in future from a new server

Daniel; 2017-10-14