Geodetic Reference Systems

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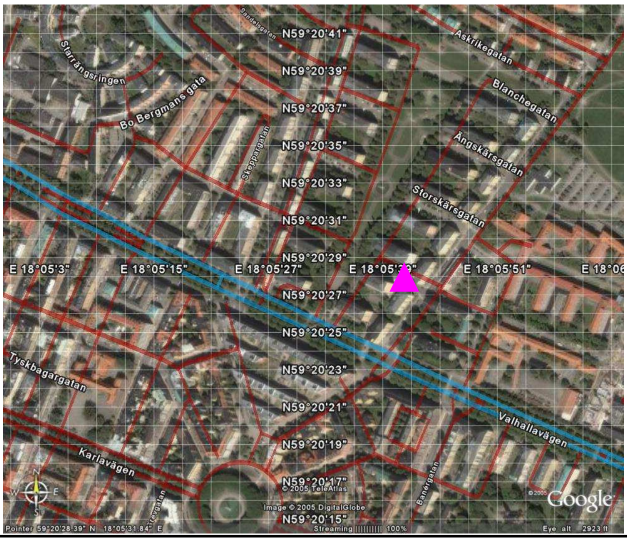
GRS overview

- Why? What is new?
- Astrogeodetic triangulation (2D)
- Height systems from levelling (1D)
- 3D geocentric reference systems
- Swedish geodetic infrastructure

Stockholm in Google Earth (2005)



Satellite images are in **WGS 84**

Street lines (**red** or **blue** lines) are in Stockholm city's **local coordinate system**

An old subject becomes a new topic

- High (10^{3}) accuracies of space geodetic methods (GNSS, VLBI, SLR, DORIS)
  - individual measurements more accurate than the system
  - astro-geodynamic effects become significant
- Needs for 3D reference systems, instead of 2+1D
- Globalization of positioning and mapping
- Widely use of geospatial databases and GIS
- Transition period for both old and new GRS
- How to make levelling effective or replace it?

Reference system vs reference frame

- Reference system: theoretical definition of the origin, scale, orientation of the 3 coordinate axes, together with related mathematical and physical assumptions
- Reference frame: a network of ground points or celestial bodies, at which the coordinates and velocities in a certain reference system have been explicitly specified, as a realization of the theoretical reference system

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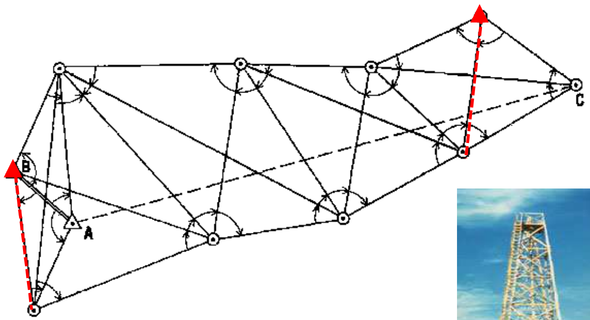
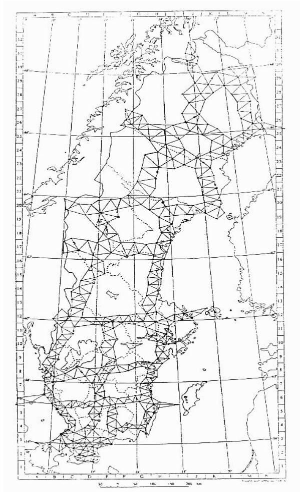
- Geodetic infrastructure in Sweden

Astrogeodetic triangulation





General procedures

- Select a reference ellipsoid and define a *datum*
- Design a network of triangles
- Astronomical observations (*a fewer*)
- Geodetic measurements (angles, distances etc)
- Reduction of ground meaurements to the ellipsoid
- Least squares adjustment on the ellipsoid
- Define an official 2D coordinate system
- Re-estimation of new ellipsoidal parameters

Reduction of astronomical coordinates

$$\phi = \Phi - \xi$$
$$\lambda = \Lambda - \eta / \cos\phi$$

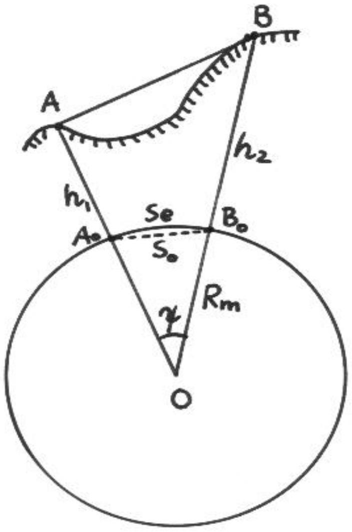
$$\alpha_{ab} = A_{ab} - \Delta A_1 - \Delta A_2$$

$$\left.\begin{array}{l} \Delta A_1 = \eta \; tg\phi = (\Lambda - \lambda)\sin\phi \\ \Delta A_2 = (\xi \sin\alpha_{ab} - \eta\cos\alpha_{ab})\cot z_{ab} \end{array}\right\}$$

When z_{ab} ~90° , we get the **Laplace condition**:

$$A_{ab} - \alpha_{ab} = (\Lambda - \lambda)\sin\phi$$

Reduction of slope distances

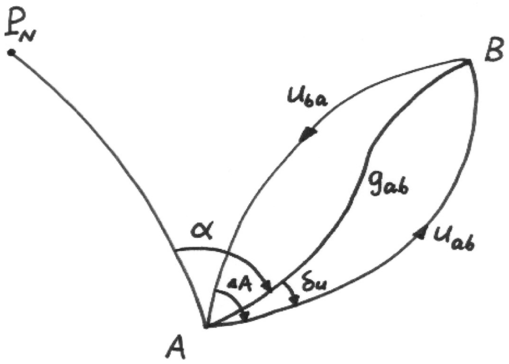


$$s_0 = \sqrt{\frac{s^2 - (h_2 - h_1)^2}{\left(1 + \frac{h_1}{R_m}\right)\left(1 + \frac{h_2}{R_m}\right)}}$$

$$R_m = \frac{a\sqrt{1-e^2}}{W^2} = \frac{a\sqrt{1-e^2}}{1 - e^2\sin^2\phi_m}$$

$$s_e = 2R_m \cdot \arcsin\left(\frac{s_0}{2R_m}\right)$$

Reduction of ground directions (angles)



- Reduction due to deflection of the vertical

$$A_{ab} - \alpha_{ab} = (\Lambda - \lambda)\sin\phi$$

- Reduction due to discrepancy between normal section and geodesic

$$\delta_u \approx \Delta A/3 \approx \frac{e^2 \cdot s^2}{12\ N^2}\cos^2\phi\ \sin 2\alpha$$

- Reduction due to height of the object above the ellipsoid

$$\delta_h \approx -\frac{h}{2N}\, e'^2\ \cos^2\phi\ \sin 2\alpha$$

Least squares adjustment on the ellipsoid

- **Unknown parameters to be determined:** geodetic latitudes and longitudes of unknown triangulation points
- **Observations:** distances, azimuths, angles/directions
- **Observation equation:** an explicit function of the unknown parameters for each observation
- **If an observation equation is non-linear:** the observation equation must be linearized using approximate geodetic latitudes/longitudes

Observation equation of a distance

$$s_{ij} - \varepsilon_{ij} = s(\phi_i^0 + \delta\phi_i,\ \lambda_i^0 + \delta\lambda_i,\ \phi_j^0 + \delta\phi_j,\ \lambda_j^0 + \delta\lambda_j)$$

$$\approx s_{ij}^0 + \frac{\partial s}{\partial \phi_i} \cdot \delta\phi_i + \frac{\partial s}{\partial \lambda_i} \cdot \delta\lambda_i + \frac{\partial s}{\partial \phi_i} \cdot \delta\phi_j + \frac{\partial s}{\partial \lambda_i} \cdot \delta\lambda_j$$

$$\ell_{ij} - \varepsilon_{ij} = a_i \cdot \delta\phi_i + b_i \cdot \delta\lambda_i + a_j \cdot \delta\phi_j + b_j \cdot \delta\lambda_j$$

$$\begin{aligned} a_i &= -M_i^0 \cos\alpha_{ij}^0 \\ b_i &= N_j^0 \cos\phi_j^0 \sin\alpha_{ji}^0 \\ a_j &= -M_j^0 \cos\alpha_{ji}^0 \\ b_j &= -N_j^0 \cos\phi_j^0 \sin\alpha_{ji}^0 \\ \ell_{ij} &= s_{ij} - s_{ij}^0 \end{aligned}$$

For linearization:

$$\begin{aligned} \phi_i &= \phi_i^0 + \delta\phi_i \\ \lambda_i &= \lambda_i^0 + \delta\lambda_i \\ \phi_j &= \phi_j^0 + \delta\phi_j \\ \lambda_j &= \lambda_j^0 + \delta\lambda_j \end{aligned}$$

$$\begin{aligned} s_{ij}^0 &= s(\phi_i^0,\ \lambda_i^0,\ \phi_j^0,\ \lambda_j^0) \\ \alpha_{ij}^0 &= \alpha(\phi_i^0,\ \lambda_i^0,\ \phi_j^0,\ \lambda_j^0) \end{aligned}$$

Observation equation of an azimuth

$$\alpha_{ij} - \varepsilon_{ij} = \alpha(\phi_i^0 + \delta\phi_i,\ \lambda_i^0 + \delta\lambda_i,\ \phi_j^0 + \delta\phi_j,\ \lambda_j^0 + \delta\lambda_j)$$

$$\approx \alpha_{ij}^0 + \frac{\partial \alpha}{\partial \phi_i} \cdot \delta\phi_i + \frac{\partial \alpha}{\partial \lambda_i} \cdot \delta\lambda_i + \frac{\partial \alpha}{\partial \phi_j} \cdot \delta\phi_j + \frac{\partial \alpha}{\partial \lambda_j} \cdot \delta\lambda_j$$

$$\ell_{ij} - \varepsilon_{ij} = c_i \cdot \delta\phi_i + d_i \cdot \delta\lambda_i + c_j \cdot \delta\phi_j + d_j \cdot \delta\lambda_j$$

$$\left.\begin{aligned} c_i &= M_i^0 \sin\alpha_{ij}^0 / s_{ij}^0 \\ d_i &= N_j^0 \cos\phi_j^0 \cos\alpha_{ji}^0 / s_{ij}^0 \\ c_j &= M_j^0 \sin\alpha_{ji}^0 / s_{ij}^0 \\ d_j &= -N_j^0 \cos\phi_j^0 \cos\alpha_{ji}^0 / s_{ij}^0 \\ \ell_{ij} &= \alpha_{ij} - \alpha_{ij}^0 \end{aligned}\right\}$$

Least squares adjustment on the ellipsoid

Final observation equations for all measurements

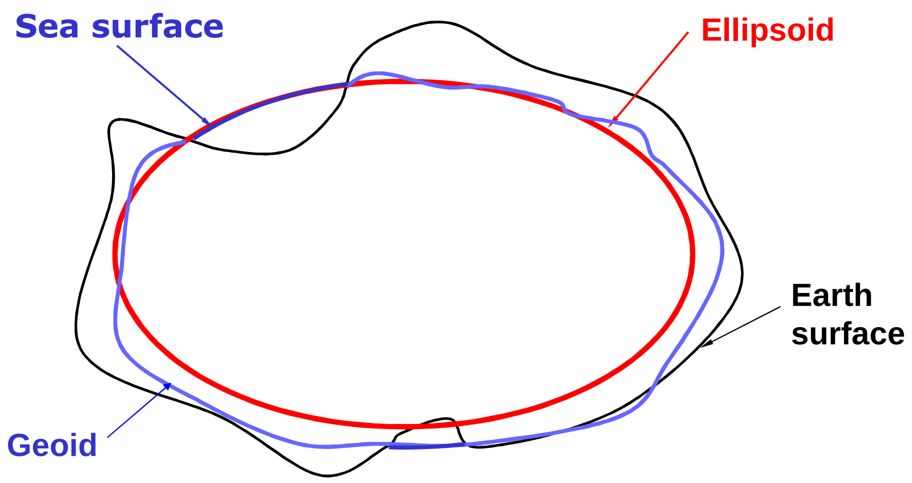
$$L - \varepsilon = A\,X$$

L : (reduced) measurements (azimuths, angles, distances etc)
ε : residuals (errors)
X : unknowns (coordinate corrections, other parameters)
A : design matrix

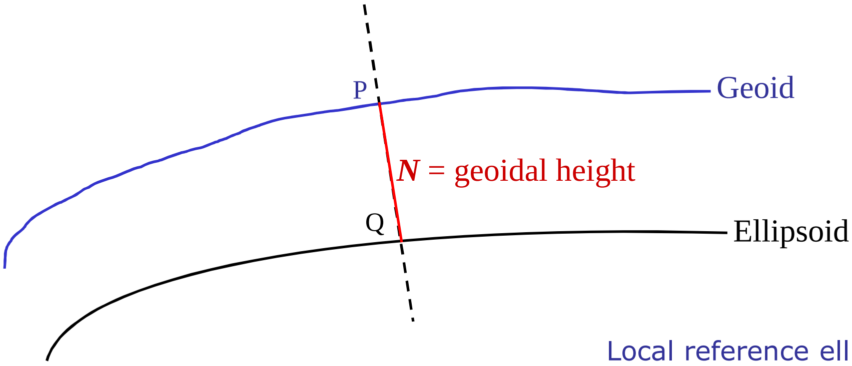
Least squares solution:

$$\hat{X} = \left(A^\top P A\right)^{-1} A^\top P L$$

Fitting ellipsoids to the geoid



The geoid-ellipsoid separation



Mean Earth Ellipsoid (σ=whole earth)

$$\iint_{\sigma} N^2 d\sigma = \text{minimum}$$

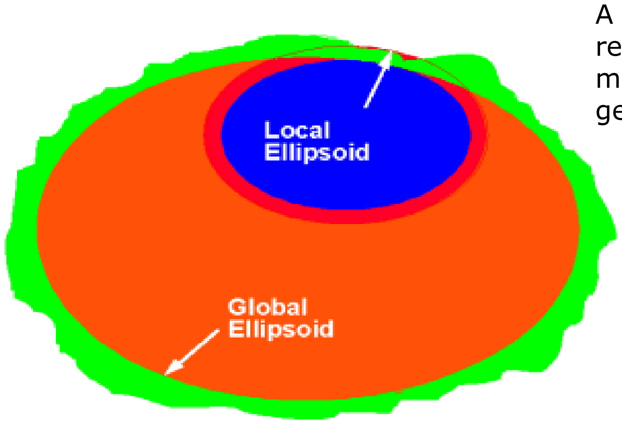
Local reference ellipsoid

(σ_1 = local area, a country)

$$\iint_{\sigma_1} N^2 d\sigma = \text{minimum}$$

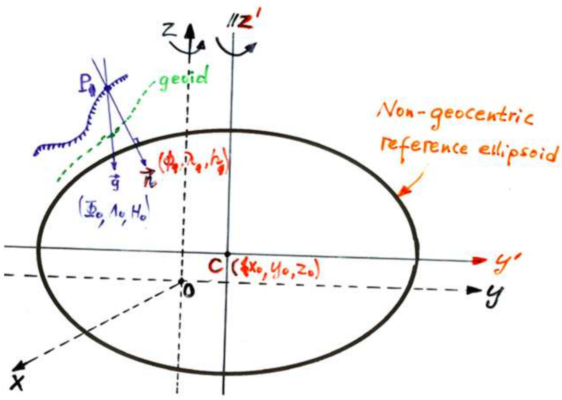
Local vs global ellipsoids

A locally fitted reference ellipsoid is most often not geocentric



A globally fitted reference ellipsoid (*Mean Earth Ellipsoid*) is geocentric

Positioning of the ellipsoid



O: geocentre
C: centre of the reference ellipsoid

Assumption: ellipsoid's minor axis Z' is parallel to the Earth rotation axis Z

→ the equator of the ellipsoid is parallel to the Earth's equator

→ position of the ellipsoid in relation to the earth can be defined by the geocentric coordinates (x_0,y_0,z_0) of the ellipsoidal center C

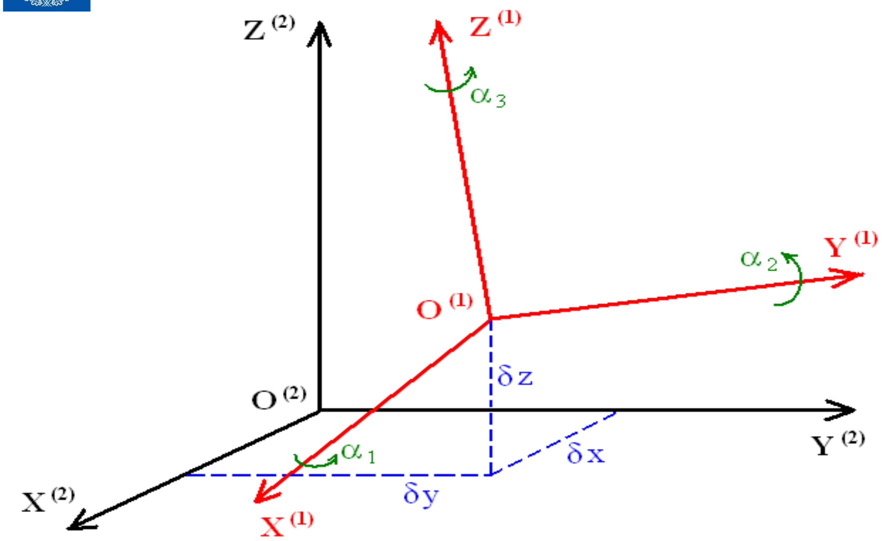
Geodetic datum (*ellipsoidal datum*)

- A *geodetic datum* consists of :
  - reference ellipsoid (size, shape and position defined) **and**
  - a set of ground triangulation points whose 2D geodetic coordinates (φ,λ) are computed with respect to this reference ellipsoid
- Assumptions:
  - The minor axis of ellipsoid is parallel to earth rotation axis;
  - The initial median plane of the ellipsoid is parallel to the *astronomical* Greenwich meridian plane
- The position of the ellipsoid is often/best defined by the geocentric coordinates of the ellipsoidal centre (x_0, y_0, z_0)

Problems with astrogeodetic systems

- National/regional astrogeodetic systems are often non-geocentric
- Different ellipsoids and map projections in use
- Accuracy of triangulation coordinates are lower than modern GNSS
- Transformation between triangulation- and GNSS-based coordinate systems is needed during transition period
- Methods, procedures and services for coordinate transformation should be developed/provided.

Two 3D coordinate systems



Helmert model of 3D transformation

$$\begin{bmatrix} X_i^{(2)} \\ Y_i^{(2)} \\ Z_i^{(2)} \end{bmatrix} = \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} + s \cdot R(\alpha_1, \alpha_2, \alpha_3) \cdot \begin{bmatrix} X_i^{(1)} \\ Y_i^{(1)} \\ Z_i^{(1)} \end{bmatrix}$$

$$s = 1 + \delta s$$

$$\underset{3\times 3}{R} = R(\alpha_1, \alpha_2, \alpha_3) = R_3(\alpha_3) \cdot R_2(\alpha_2) \cdot R_1(\alpha_1)$$

$$= \begin{bmatrix} \cos\alpha_3 & \sin\alpha_3 & 0 \\ -\sin\alpha_3 & \cos\alpha_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\alpha_2 & 0 & -\sin\alpha_2 \\ 0 & 1 & 0 \\ \sin\alpha_2 & 0 & \cos\alpha_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_1 & \sin\alpha_1 \\ 0 & -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha_2\cos\alpha_3 & \cos\alpha_1\sin\alpha_3 + \sin\alpha_1\sin\alpha_2\cos\alpha_3 & \sin\alpha_1\sin\alpha_3 - \cos\alpha_1\sin\alpha_2\cos\alpha_3 \\ -\cos\alpha_2\sin\alpha_3 & \cos\alpha_1\cos\alpha_3 - \sin\alpha_1\sin\alpha_2\sin\alpha_3 & \sin\alpha_1\cos\alpha_3 + \cos\alpha_1\sin\alpha_2\sin\alpha_3 \\ \sin\alpha_2 & -\sin\alpha_1\cos\alpha_2 & \cos\alpha_1\cos\alpha_2 \end{bmatrix}$$

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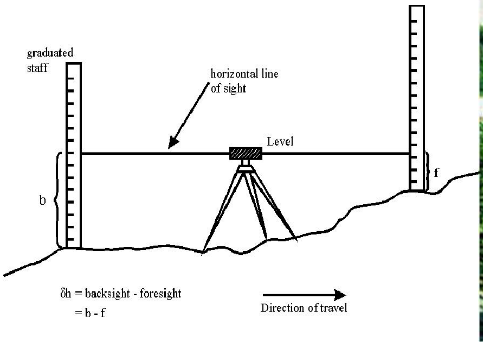
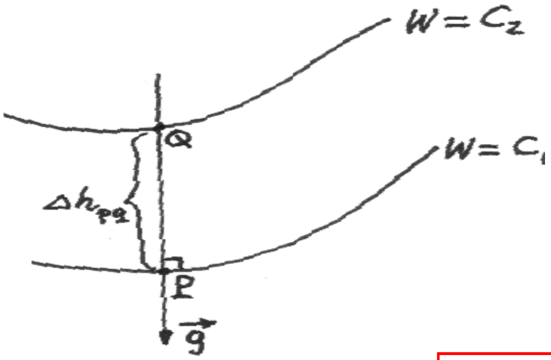
Height differences from levelling



Swedish Motorized Levelling

Distance between two close equipotential surfaces



$$g = |\vec{g}| = -\frac{dW}{dn}$$

$$g_p \approx -\Delta W_{pq} / \Delta h_{pq}$$

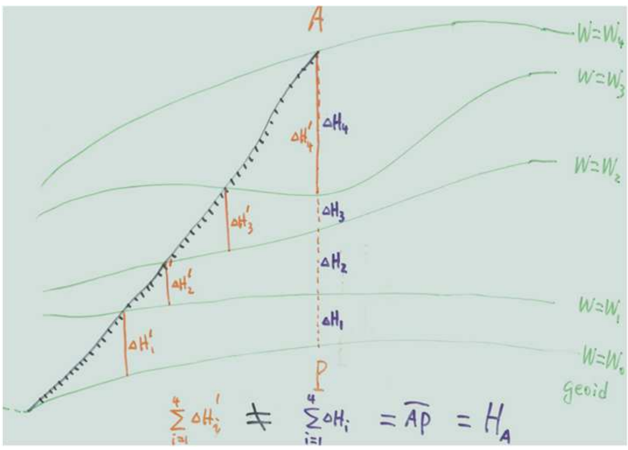
$$\Delta h_{pq} \approx -\Delta W_{pq} / g_p$$

$\longrightarrow$ Equipotential surfaces are not parallel !!!

$$\Delta W_{pq} = W_q - W_p = C_2 - C_1 = constant$$

Theoretical misclosure (ε) of levelling

$\longrightarrow$ Equipotential surfaces are not parallel !!!



Levelling between two benchmarks along 2 different routes show different height differences

Levelling along a closed loop is not equal to zero:

$$\oint dh = \varepsilon \neq 0;$$

$\longrightarrow$ ε: theoretical misclosure

Height Systems

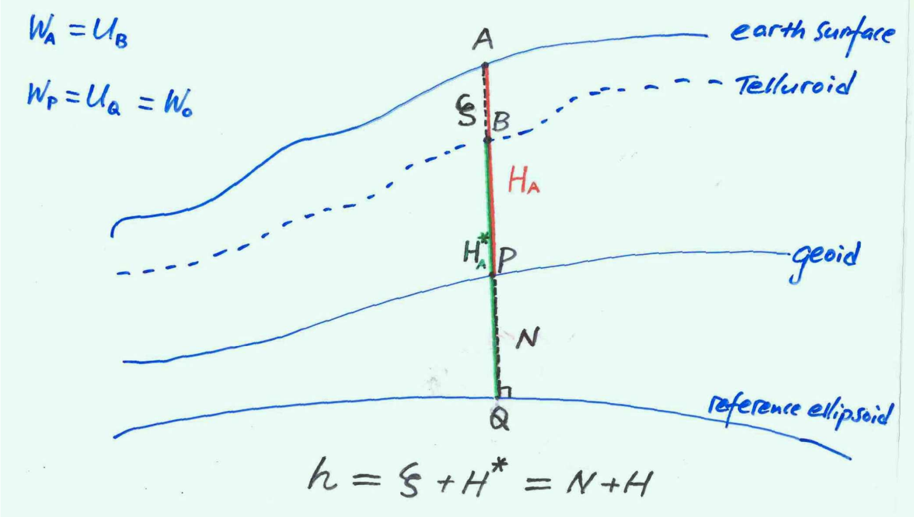
Geopotential number of point A: $C_A = W_0 - W_A$

Orthometric height of point A: $H_A = \frac{C_A}{\bar{g}_A} = \frac{W_0 - W_A}{\bar{g}_A}$

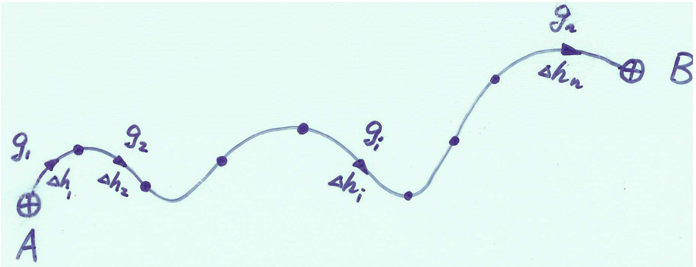
Normal height of point A: $H_A^* = \frac{C_A}{\bar{\gamma}_A} = \frac{W_0 - W_A}{\bar{\gamma}_A}$

Dynamic height of point A: $H_A^d = \frac{C_A}{\gamma_0} = \frac{W_0 - W_A}{\gamma_0}$

Concepts of Height Systems



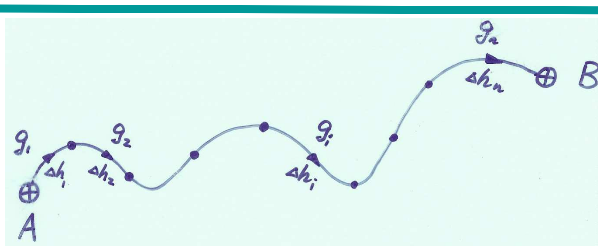
Principle of Precise levelling



Given: H_A^*, g_i, Δh_i $(i = 1, 2, 3, \dots n)$

Sought: $H_B^* = ?$

Principle of Precise levelling



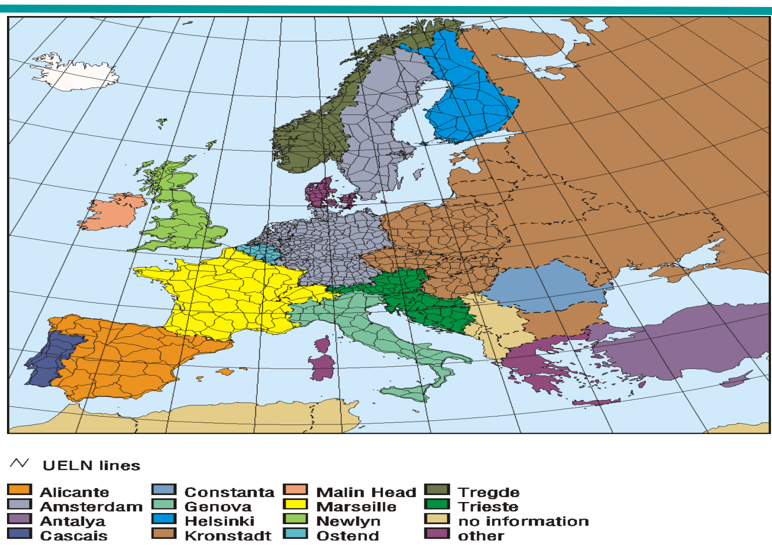
Solution:

1. $C_A = H_A^* \cdot \bar{\gamma}_A$
2. $\Delta C_{AB} \triangleq C_B - C_A = \sum_{i=1}^{n} (g_i \cdot \Delta h_i)$
3. $C_B = C_A + \Delta C_{AB}$
4. $H_B^* = \frac{C_B}{\bar{\gamma}_B}$

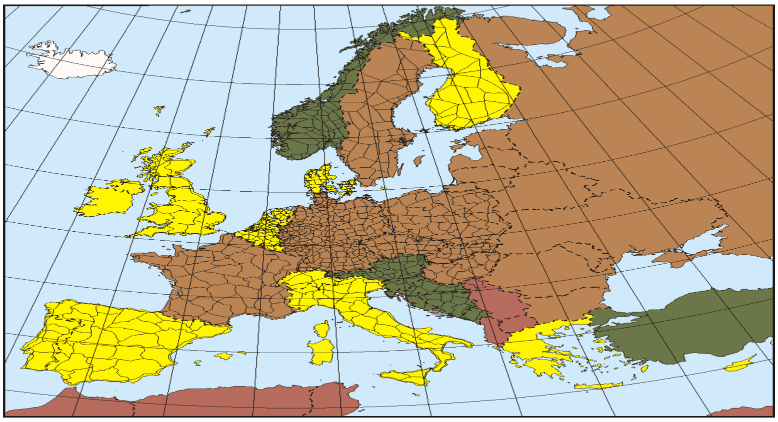
Problems with height systems

- Orthometric heights are not possible to determine. Only geopotential differences can be uniquely determined. Gravity data is needed !
- Different height systems in use
- Different zero datums
- Local height systems are not connected.

Zero height datums in Europe



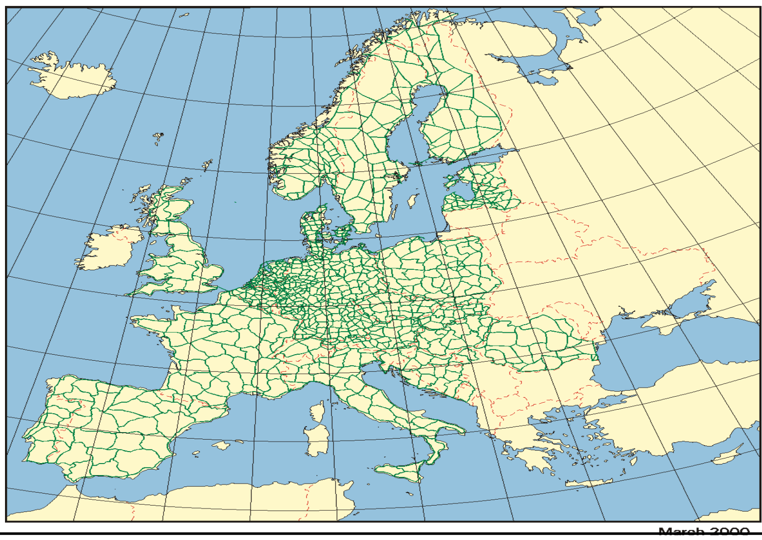
Height systems in Europe



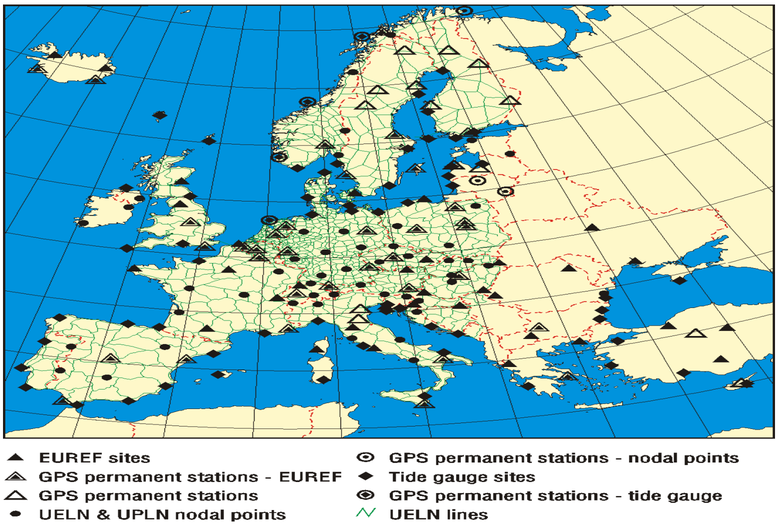
Kind of Heights

- normal heights
- orthometric heights
- normal orthometric heights
- no information
- no levelling heights
- UELN lines

United European Levelling Network (UELN95/98)



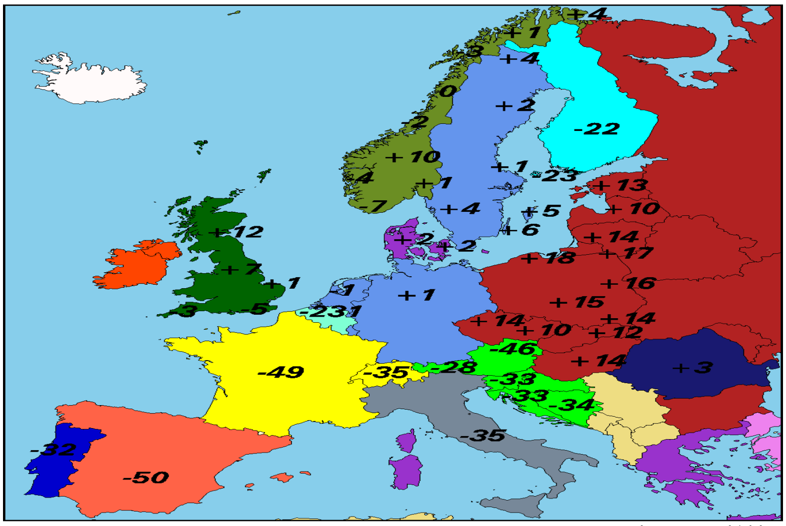
EUVN – EUropean Vertical GPS Network



European Vertical Reference System 2000

- United European Levelling Network
  - Only levelling lines between nodal points are included
  - Different realizations of the mean sea level
  - Different height systems
- EUropean Vertical GPS Network
  - Connect different height system in one common system using GPS
  - Including: 66 EUREF GPS stations, 54 UELN nodal points and 63 Mareograph/tide gauge stations
- **European Vertical Reference System 2000 (EVRS 2000)**
  - Zero datum at Amsterdam: $W_0=(W_0)NAP=(U_0)_{GRS80}$
  - Heights expressed as geopotential numbers. Normal heights used
  - Zero tidal system

Height Differences: National vs EVRS 2000



September 2000

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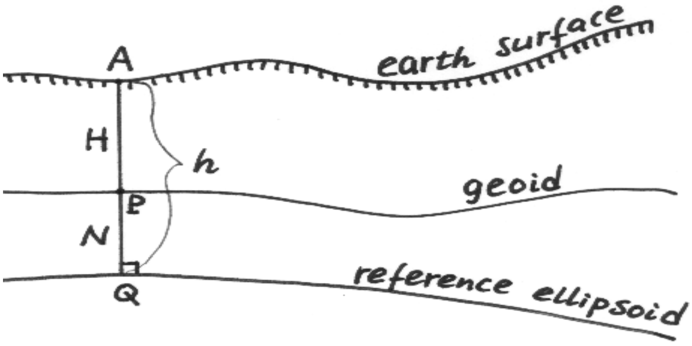
- GNSS-derived ellipsoidl heights are not related to the geoid (MSL).

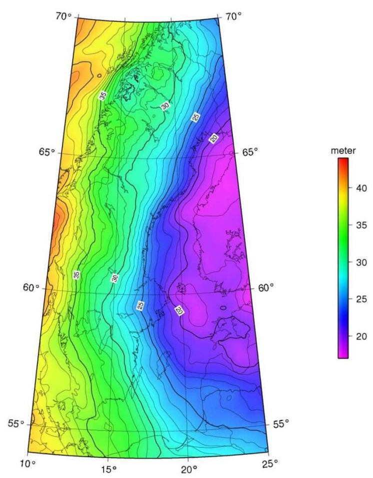
Ellipsoidal heights vs heights above MSL

h=H+N



- GPS survey provides ellipsoidal heights (h)
- Mapping and construction requires heights above the geoid (H)
- To know the geoid heights (N), one has to know the gravity field

Gravimetric Geoid Model - SWEN 08LR



- Computed using the KTH method by modifying Stokes' formua and combining global gravitational models with terrestrial gravity data.
- Fitted to 1570 GPS/levelling points (SWEREFF 99 / RH 2000)
- Remaining residuals are smoothly interpolated
- A height transformation model from SWEREF 99 to RH 2000
- Accuracy:
  - 1 – 1.5 cm in most areas
  - 5 – 10 cm in Northwest without levelling

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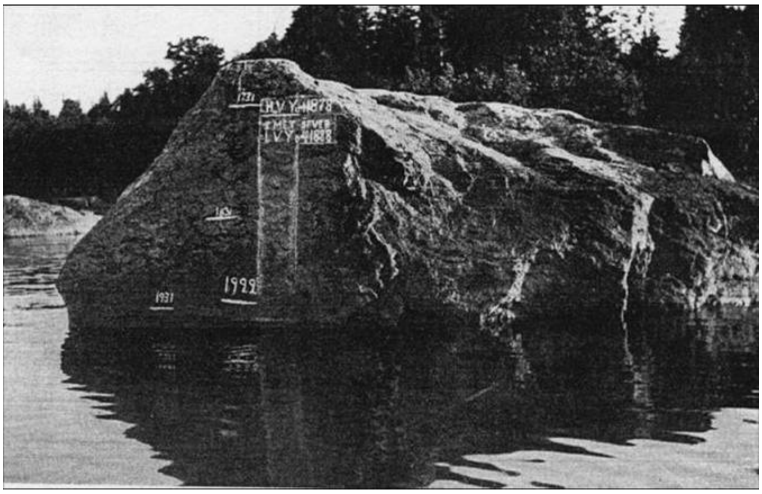
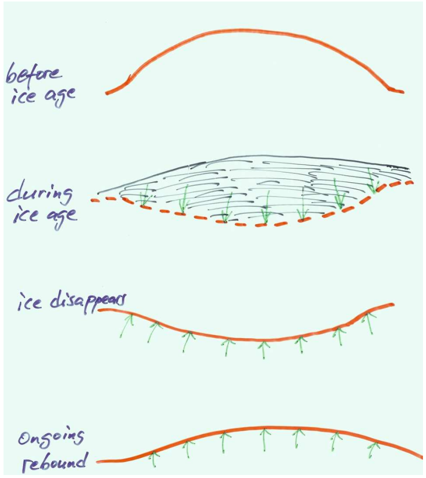
Land uplift or water sinking ???

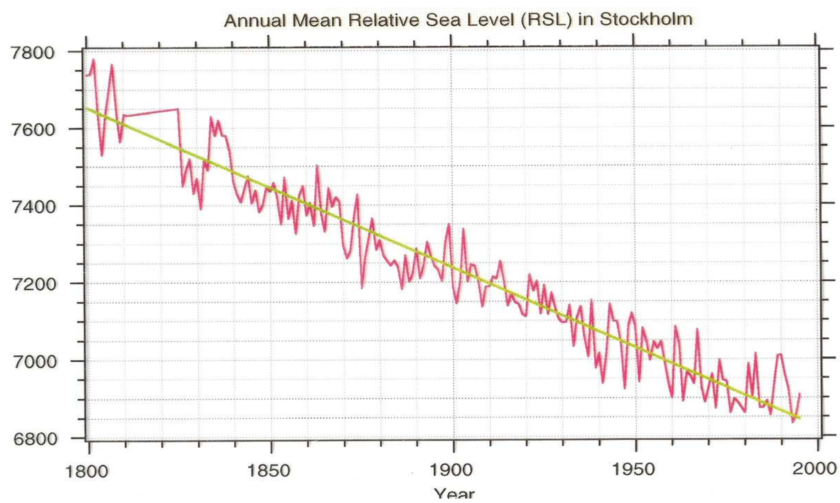


Anders Celsius 1765

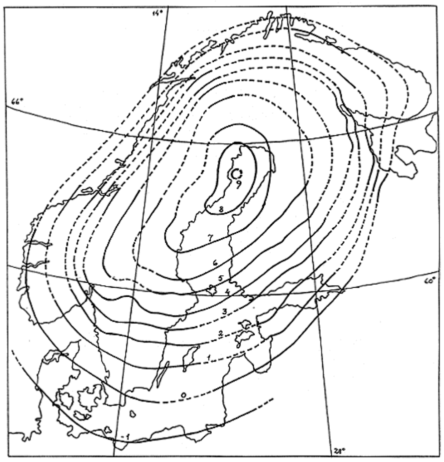
Land uplift - postglacial rebound



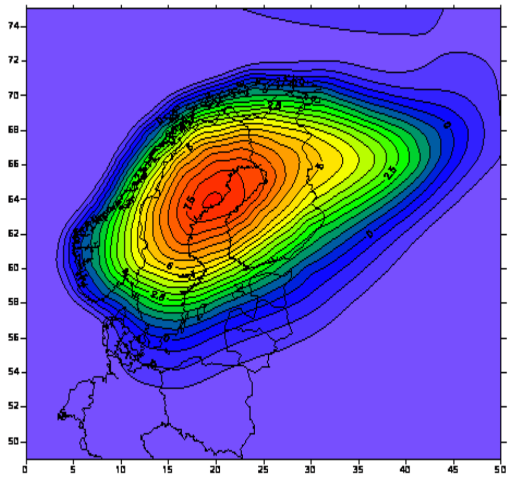
Relative sea level at tide gauge



Swedish land uplift model (mm/year)



Martin Ekman (1998)



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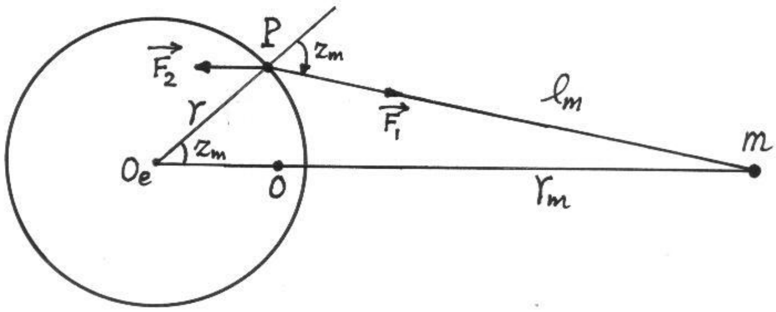
- Vertical crustal movement affects heights
- Treatment of permanent tide in height definition

Tidal potential of the Moon



$$T^m = \frac{Gm}{\ell_m} - \left(\frac{Gm}{r_m} + Gm\frac{r}{r_m^2}\cos z_m \right)$$

Non-zero time-average

$$T_2 = D \left[\left(\frac{1}{3} - \sin^2\phi \right) + 3\left(\sin^2\phi - \frac{1}{3} \right) \sin^2\delta + \cos^2\phi \cos^2\delta \cos 2t + \sin 2\phi \sin 2\delta \cos t \right]$$

$$T_2 = D \ (T_z + T_s + T_t)$$

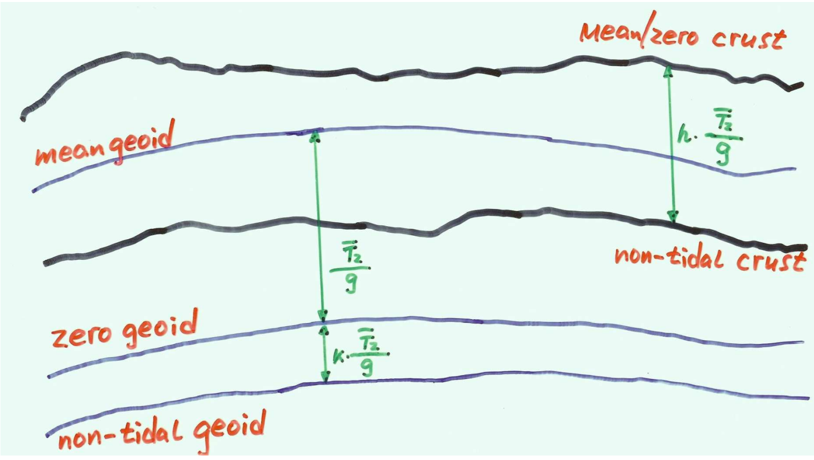
$$\overline{T_s} = \frac{1}{2\pi}\int_0^{2\pi} D \cdot T_s \, dt = 0$$
$$\overline{T_t} = \frac{1}{2\pi}\int_0^{2\pi} D \cdot T_t \, dt = 0$$
$$\overline{T_z} = \frac{1}{2\pi}\int_0^{2\pi} D \cdot T_z \, d\ell = D \ \left(\tfrac{1}{2}\sin^2\varepsilon - \tfrac{1}{3}\right) \ \left(\sin^2\phi - \tfrac{1}{3}\right)$$

The time-average of the tidal potential T_2 does not vanish. This causes permanent tidal effects, including permanent deformation of the solid earth.

Permanent tide and tidal systems

- 4 parts of tidal effects
  - Direct (extra-terrestrial masses) vs indirect effects (earth's deformation)
  - Periodic vs permanent effetcs. Periodic effects can be averaged out.
  - Treatment of the permanent tide leads to 3 tidal systems
- Non-tidal model (Tide-free) (GNSS - ITRF)
  - Permanent deformation of the earth is *eliminated* ⟶ *earth's form changed*
  - Permanent tidal effects are *eliminated* from the measurements
- Zero tidal model (EVRS 2000)
  - Permanent deformation of the earth is *retained* ⟶ *masses outside the geoid*
  - Permanent tidal effects are *eliminated* in the measurements
- Mean tidal model
  - Permanent deformation of the earth is *retained*
  - Permanent tidal effects are *retained* ⟶ *included in the normal gravity filed ?)*

Tidal Systems: Non-Tidal, Mean vs Zero Tide



Tidal Systems: geoidal differences

Latitude FI	3 sin(FI) **2 - 1	Geoid separation, cm	Geoid separation, cm	Crust separation, cm					
(degree)		*Non-tidal - Zero*	*Zero - Mean*	*Non-tidal - Mean/zero*					
0	-1,000000000	6,80	2,04	4,21	PI/180	0,017453293	D =	26206	cm**2/s**2
5	-0,977211630	6,64	1,99	4,12	YPSILON, deg	23,439291	g=	980	cm/s**2
10	-0,909538931	6,18	1,85	3,83	sin (YPSILON)	0,3977772	D/g	26,741	cm
15	-0,799038106	5,43	1,63	3,37			T0	-6,79805	
20	-0,649066665	4,41	1,32	2,74					
25	-0,464181415	3,16	0,95	1,96					
30	-0,250000000	1,70	0,51	1,05					
35	-0,013030215	0,09	0,03	0,05					
40	0,239527733	-1,63	-0,49	-1,01					
45	0,500000000	-3,40	-1,02	-2,11					
50	0,760472267	-5,17	-1,55	-3,21					
55	1,013030215	-6,89	-2,07	-4,27					
60	1,250000000	-8,50	-2,55	-5,27					
65	1,464181415	-9,95	-2,99	-6,17					
70	1,649066665	-11,21	-3,36	-6,95					
75	1,799038106	-12,23	-3,67	-7,58					
80	1,909538931	-12,98	-3,89	-8,05					
85	1,977211630	-13,44	-4,03	-8,33					
90	2,000000000	-13,60	-4,08	-8,43					

GRS overview

- Why? What is new?
- Astrogeodetic triangulation (2D)
- Height systems from levelling (1D)
- 3D geocentric reference systems
- Geodetic infrastructure in Sweden

Celestial vs terrestrial reference system

- Celestial reference system: a reference system which is fixed in space with respect to stars and other planets
- Terrestrial reference system: a reference system which is fixed with respect to the earth's body

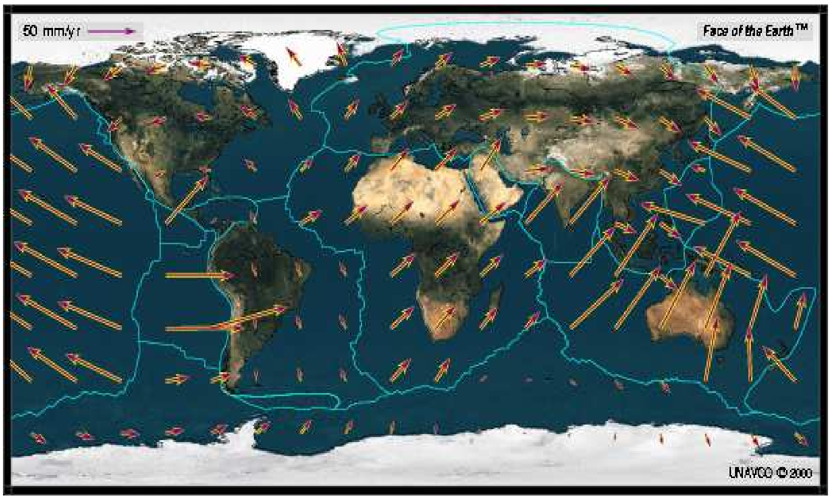
Astro-geodynamic effects

- Global tectonic movement (*continental drift*)

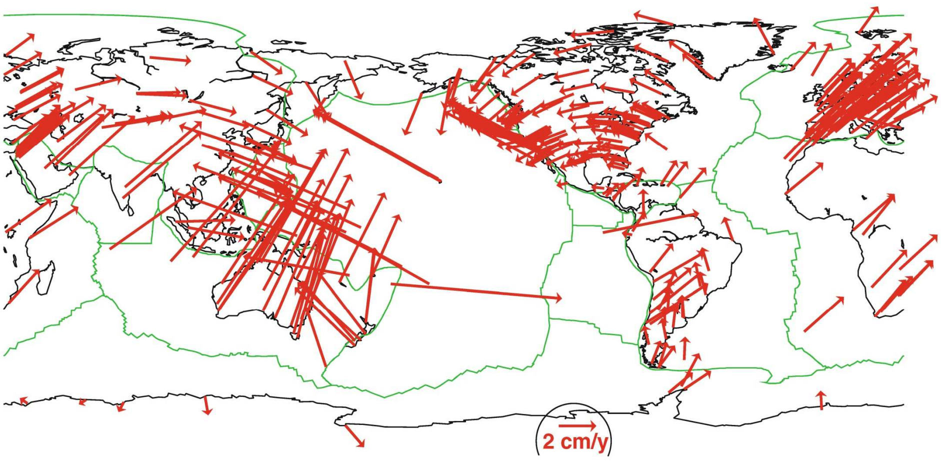
  ⟶ time-dependent coordinates: TRF *xxxx*.
  coordinates + velocities: $x, y, z, \dot{x}, \dot{y}, \dot{z}$

- Changes in earth rotation
  - Polar motion → affecting TRS
  - Precession and nutation → affecting CRS
  - Length of Day (LoD) → affecting CRS and TRS

Tectonic plate model NNR NUVEL-1A

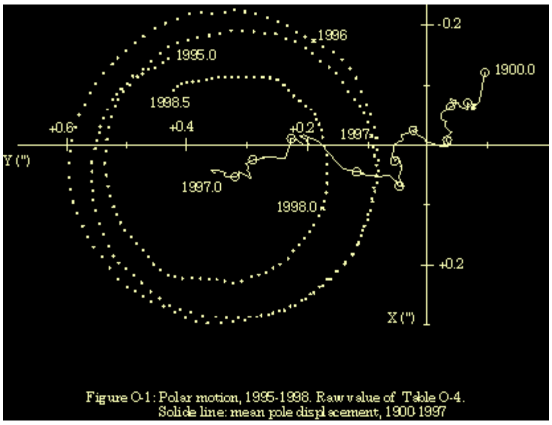


Horizontal velocity from ITRF 2008



406 stations have formal errors < 0.2mm/year

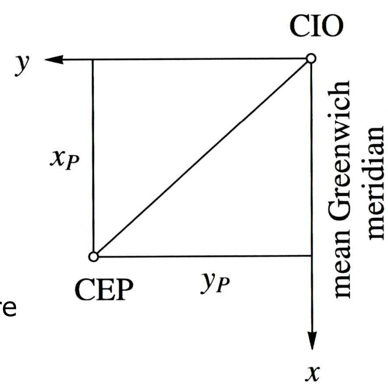
Polar motion



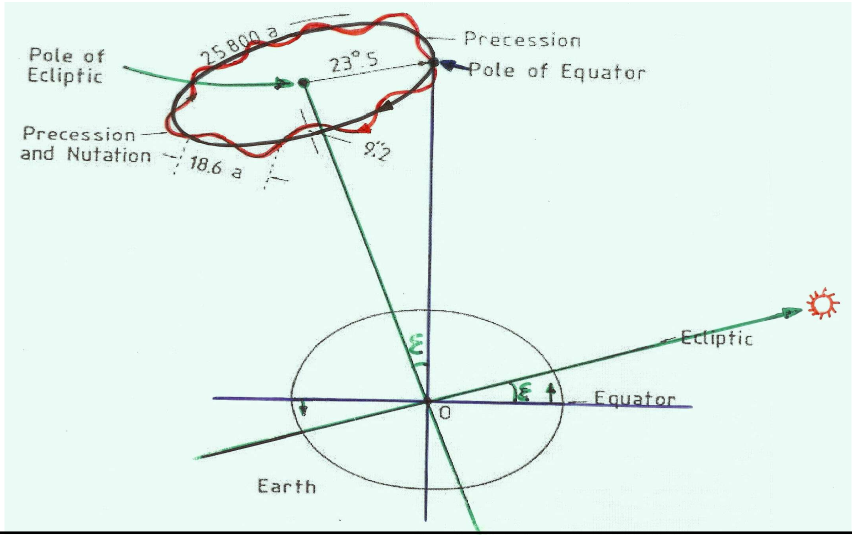
- Euler period: 305 days
- Chandler period: 430d
- Approx. radius: 8m
- Daily radius: 0,5m

Conventional International Origin (CIO)

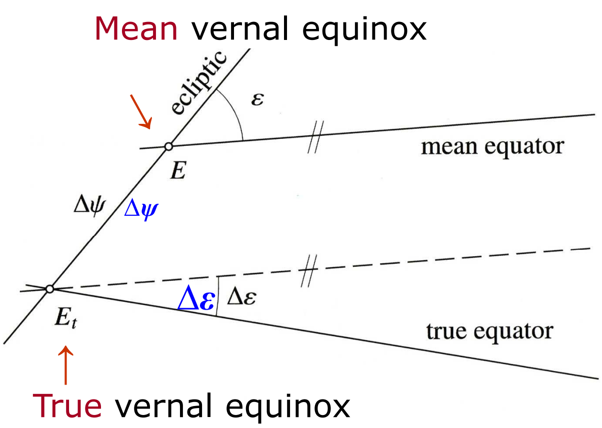
- CIO = mean position of the North Pole 1900-1905
- The instantaneous celestial ephemeris pole (CEP) is defined by polar coordinates (x_P, y_P)
- Convention: measurements w.r.t instantaneous CEP, while coordinates are defined w.r.t. CIO



Precession and nutation



Nutation parameters



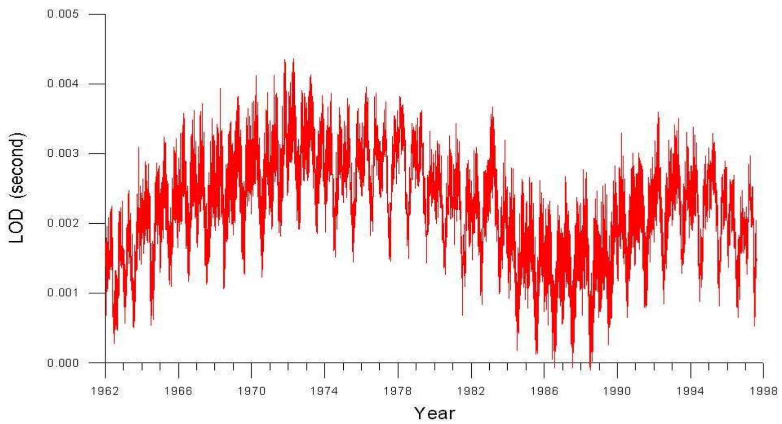
Nutation in longitude:

$\Delta\psi$

Nutation in obliquity:

$\Delta\varepsilon$

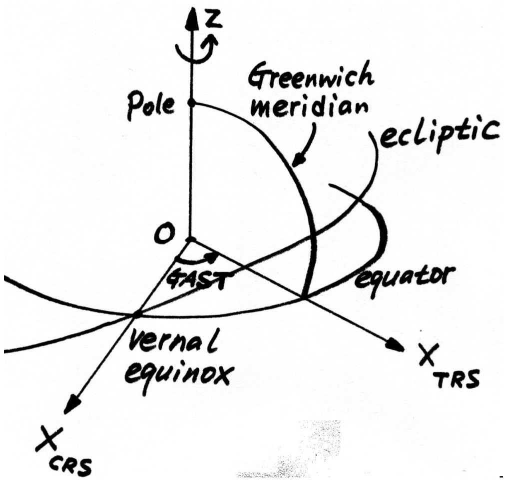
Variation in Length of Day (LoD)



Earth Orientation Parameters (EOP)

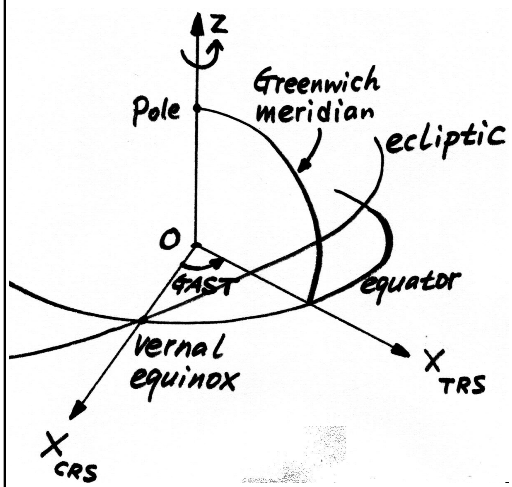
- 2 Polar coordinates with respect to **CIO** : x_p , y_p
- 2 Nutation parameters :
  - nutation in obliquity ($\Delta\varepsilon$)
  - nutation in longitude ($\Delta\psi$)
- 1 time parameter
  - UTC-UT1 (provide UT1 when UTC is known)

International Celestial Reference System (ICRS)



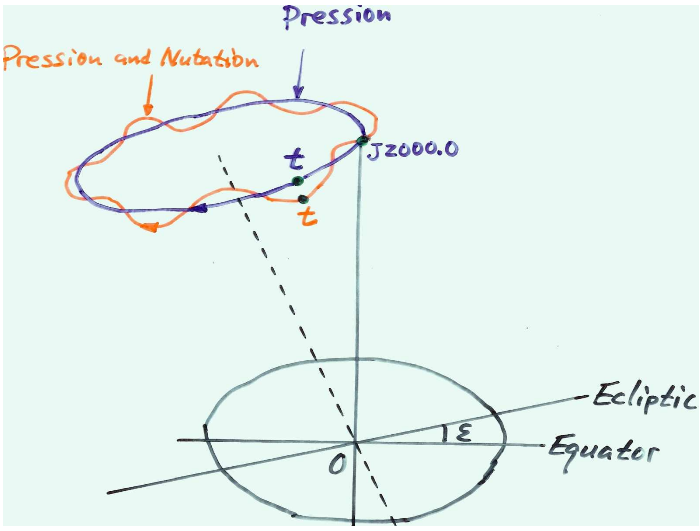
- Origin at ***barycentre*** (practically at geocentre)
- Z-axis toward the ***mean celestial North Pole* at J2000.0**
- X-axis toward the ***mean equinox* at J2000.0**
- Y-axis follows to form a right-handed system

International Terrestrial Reference System (ITRS)



- Origin at the earth's centre of masses including oceans and atmosphere
- Z-axis toward the **CIO**
- X-axis along intersection of the equator and the **Greenwich Meridian**
- Y-axis inside the equator to form a right-handed system

Reduction of precession and nutation



Transformation from ICRS to ITRS

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_p\left(x_p, y_p\right) \cdot R_3\left(GAST\right) \cdot N\left(\varepsilon_0, \Delta\varepsilon, \Delta\psi\right) \cdot P\left(z, \vartheta, \xi\right) \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

ITRS ***ICRS***

$$P\left(z, \vartheta, \xi\right) = R_3(-z) \cdot R_2(\vartheta) \cdot R_3(-\xi)$$

$$= \begin{bmatrix} +\cos z \cos\vartheta \cos\xi - \sin z \sin\xi & -\cos z \cos\vartheta \sin\xi - \sin z \cos\xi & -\cos z \sin\vartheta \\ +\sin z \cos\vartheta \cos\xi + \cos z \sin\xi & -\sin z \cos\vartheta \sin\xi + \cos z \cos\xi & -\sin z \sin\vartheta \\ +\sin\vartheta \cos\xi & -\sin\vartheta \sin\xi & +\cos\vartheta \end{bmatrix}$$

$$\underset{3\times 3}{N}\left(\varepsilon_0, \Delta\varepsilon, \Delta\psi\right) = R_1\left(-\varepsilon\right) \cdot R_3\left(-\Delta\psi\right) \cdot R_1\left(\varepsilon_0\right) \qquad (\ \varepsilon = \varepsilon_0 + \Delta\varepsilon\ )$$

$$= \begin{bmatrix} \cos\Delta\psi & -\sin\Delta\psi \cos\varepsilon_0 & -\sin\Delta\psi \sin\varepsilon_0 \\ \sin\Delta\psi \cos\varepsilon & +\cos\Delta\psi \cos\varepsilon \cos\varepsilon_0 + \sin\varepsilon \sin\varepsilon_0 & +\cos\Delta\psi \cos\varepsilon \sin\varepsilon_0 - \sin\varepsilon \cos\varepsilon_0 \\ \sin\Delta\psi \sin\varepsilon & +\cos\Delta\psi \sin\varepsilon \cos\varepsilon_0 - \cos\varepsilon \sin\varepsilon_0 & +\cos\Delta\psi \sin\varepsilon \sin\varepsilon_0 + \cos\varepsilon \cos\varepsilon_0 \end{bmatrix}$$

Computation of *GAST*

$$\varepsilon_0 = 23^0\ 26'\ 21.448'' - 46.8150'' \cdot T - 0.00059'' \cdot T^2 + 0.001813'' \cdot T^3$$

$$\left.\begin{aligned} \xi &= 2306.2181 \cdot T + 0.30188 \cdot T^2 + 0.017998 \cdot T^3 \\ z &= 2306.2181 \cdot T + 1.09468 \cdot T^2 + 0.018203 \cdot T^3 \\ \vartheta &= 2004.3109 \cdot T - 0.42665 \cdot T^2 - 0.041833 \cdot T^3 \end{aligned}\right\} \quad \text{(unit: arcsecond)}$$

$$T = \frac{JD(t) - J2000.0}{36525} = \frac{JD(t) - 2\ 451\ 545.0}{36525}$$

$$GAST = (GMST)_0 + 1.002\ 737\ 909\ 35 \cdot UT1 + \Delta\psi \cos\varepsilon_0$$
$$(GMST)_0 = 24\ 110.54841^s + 8\ 640\ 184.812866^s\ T_0 + 0.093\ 104^s\ T_0^2 - 6.2^s \cdot 10^{-6} T_0^3$$

$$T_0 = \frac{JD_{UT=0} - 2\ 451\ 545.0}{36525}$$

IERS

- IERS, International Earth Rotation Service, was established in 1987 by IAU and IUGG to replace
  - International Polar Motion Service (IPMS)
  - Bureau International de l'Heure (BIH)
- Renamed in 2003 as International Earth Rotation and Reference Systems Service

International Celestial Reference Frame (ICRF)

- First realization of ICRS, ICRF1, is made by IAU in 1997 based on VLBI measurements of 212 defining radio sources
- Two extentions of ICRF1 are made by IERS in 1999 and 2004
- Second realization of ICRS, ICRF2, is made in 2009
  - VLBI measurements of 3414 radio sources
  - 295 defining radio sources

International Celestial Reference Frame (ICRF2)

Coordinates of 295 ICRF2 defining sources

ICRF Designation (1)	IERS Des. (2)	Right Ascension J2000.0 h m s	Declination J2000.0 o ' "	Uncertainty R.A. s	Uncertainty Dec. "
ICRF J000435.6-473619	0002-478	00 04 35.65550384	-47 36 19.6037899	0.00001359	0.0002139
ICRF J001031.0+105829	0007+106	00 10 31.00590186	10 58 29.5043827	0.00000491	0.0000930
ICRF J001101.2-261233	0008-264	00 11 01.24673846	-26 12 33.3770171	0.00000660	0.0000936
ICRF J001331.1+405137	0010+405	00 13 31.13020334	40 51 37.1441040	0.00000482	0.0000683
ICRF J001611.0-001512	0013-005	00 16 11.08855479	-00 15 12.4453413	0.00000435	0.0001005
ICRF J001945.7+732730	0016+731	00 19 45.78641940	73 27 30.0174396	0.00000989	0.0000424
ICRF J002232.4+060804	0019+058	00 22 32.44120914	06 08 04.2690807	0.00000439	0.0000956
ICRF J003824.8+413706	0035+413	00 38 24.84359231	41 37 06.0003032	0.00000499	0.0000613
ICRF J005041.3-092905	0048-097	00 50 41.31738756	-09 29 05.2102688	0.00000278	0.0000428
ICRF J005109.5-422633	0048-427	00 51 09.50182012	-42 26 33.2932480	0.00000932	0.0001177
ICRF J010245.7+582411	0059+581	01 02 45.76238248	58 24 11.1366009	0.00000523	0.0000414
ICRF J010645.1-403419	0104-408	01 06 45.10796851	-40 34 19.9602291	0.00000376	0.0000455
ICRF J010915.4-604948	0107-610	01 09 15.47520598	-60 49 48.4599686	0.00001744	0.0001750
ICRF J011205.8+224438	0109+224	01 12 05.82471754	22 44 38.7863909	0.00000379	0.0000653
ICRF J011327.0+494824	0110+495	01 13 27.00680344	49 48 24.0431742	0.00000597	0.0000727
ICRF J011857.2-214130	0116-219	01 18 57.26216666	-21 41 30.1399986	0.00000683	0.0001138
ICRF J012141.5+114950	0119+115	01 21 41.59504339	11 49 50.4131012	0.00000279	0.0000429
ICRF J013305.7-520003	0131-522	01 33 05.76255607	-52 00 03.9457209	0.00001218	0.0001605

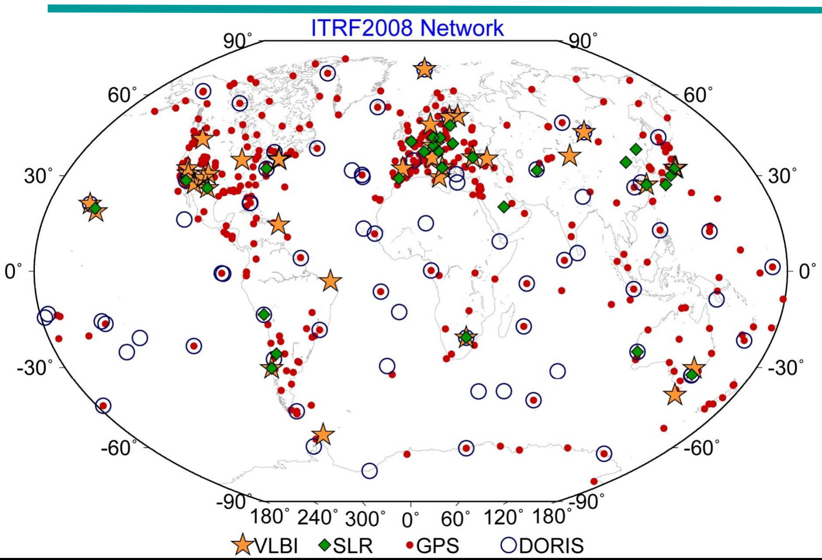
International Tererstrial Reference Frame

- ITRF is realization of ITRS
- ITRF networks consists of stations all over the world, measured by 4 space geodetic methods
  1) Very Long Baseline Interferometry (VLBI)
  2) Satellite Laser Ranging (SLR)
  3) GNSS
  4) Doppler Orbitography Integrated by Satellites (DORIS)
- Final products include:
  - Position coordinates and velocities for a reference epoch
  - Transformation parameters and their time derivatives
  - Consistent estimates of 5 EOP

List of ITRF solutions

ITRF88	1988.0 reference epoch
ITRF89	1988.0
ITRF92	1988.0
ITRF93	1988.0
ITRF94	1997.0
ITRF96	1997.0
ITRF97	1997.0
ITRF2000	1997.0
ITRF2005	2000.0
ITRF2008	2005.0
ITRF2014	2010.0

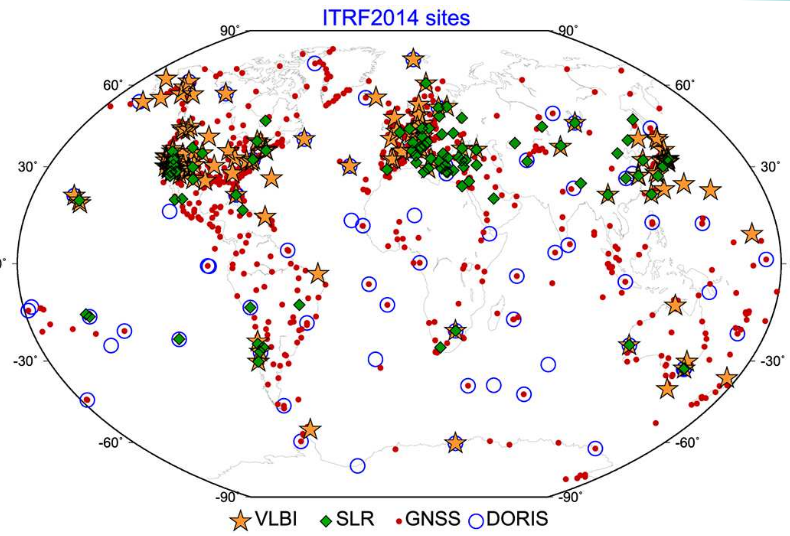
Global Network of ITRF 2008



VLBI: 138
SLR: 155
GPS: 910
DORIS: 170

Total: **934**
at 580 sites

Global Network of ITRF 2014



ITRF 2008

- Computations in 3 steps
  1) Each type (VLBI,SLR,GPS,DORIS) is processed separately to produce weekly (daily VLBI) time series of positions and EOP
  2) Above results are combined to estimate position coordinates for epoch 2005.0, velocity and daily EOP
  3) Combination with local survey at co-locating stations

- ITRF 2008 results:
  – Origin and scale are defined by SLR data
  – Orientation is defined by average of VLBI and SLR data
  – Scale accuracy: **1 ppb, 0.05 ppb/year** (VLBI-SLR agreement)
  – Origin accuracy: at the level of or better than **1 cm**

Station coordinates/velocities of ITRF 2008

10	VELX	7205	A	1	05:001:00000	m/y	2	-.154701843128797E-01	0.47543E-04
11	VELY	7205	A	1	05:001:00000	m/y	2	-.121094247184766E-02	0.62981E-04
12	VELZ	7205	A	1	05:001:00000	m/y	2	0.408529863128065E-02	0.69402E-04
13	STAX	7204	A	1	05:001:00000	m	2	0.882879772507557E+06	0.13845E-02
14	STAY	7204	A	1	05:001:00000	m	2	-.492448231271830E+07	0.35634E-02
15	STAZ	7204	A	1	05:001:00000	m	2	0.394413070534473E+07	0.27791E-02
16	VELX	7204	A	1	05:001:00000	m/y	2	-.143669405946909E-01	0.81983E-04
17	VELY	7204	A	1	05:001:00000	m/y	2	-.102511002663171E-02	0.20579E-03
18	VELZ	7204	A	1	05:001:00000	m/y	2	0.274104971026836E-02	0.17380E-03
19	STAX	7216	A	1	05:001:00000	m	2	-.132421109976204E+07	0.97620E-03
20	STAY	7216	A	1	05:001:00000	m	2	-.533202313706689E+07	0.15257E-02
21	STAZ	7216	A	1	05:001:00000	m	2	0.323211831880121E+07	0.11575E-02
22	VELX	7216	A	1	05:001:00000	m/y	2	-.126619959048342E-01	0.51526E-04
23	VELY	7216	A	1	05:001:00000	m/y	2	0.339720116768070E-03	0.84921E-04
24	VELZ	7216	A	1	05:001:00000	m/y	2	-.467559096977887E-02	0.69110E-04
25	**STAX**	**7213**	**A**	**1**	**05:001:00000**	**m**	**2**	**0.337060591485963E+07**	**0.65329E-03**
26	**STAY**	**7213**	**A**	**1**	**05:001:00000**	**m**	**2**	**0.711917602784800E+06**	**0.54467E-03**
27	**STAZ**	**7213**	**A**	**1**	**05:001:00000**	**m**	**2**	**0.534983082091565E+07**	**0.69138E-03**
28	**VELX**	**7213**	**A**	**1**	**05:001:00000**	**m/y**	**2**	**-.142016652825457E-01**	**0.54415E-04**
29	**VELY**	**7213**	**A**	**1**	**05:001:00000**	**m/y**	**2**	**0.145280633229021E-01**	**0.36261E-04**
30	**VELZ**	**7213**	**A**	**1**	**05:001:00000**	**m/y**	**2**	**0.103568368182873E-01**	**0.76909E-04**
31	STAX	7203	A	1	05:001:00000	m	2	0.403394735308593E+07	0.30487E-02
32	STAY	7203	A	1	05:001:00000	m	2	0.486990645775994E+06	0.11888E-02
33	STAZ	7203	A	1	05:001:00000	m	2	0.490043088922310E+07	0.34356E-02
34	VELX	7203	A	1	05:001:00000	m/y	2	-.137947899098904E-01	0.23546E-03

Transformation between different ITRF

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ITRFyy} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ITRF\ 2000} + \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} + \begin{bmatrix} \delta & -\omega_Z & \omega_Y \\ \omega_Z & \delta & -\omega_X \\ -\omega_Y & \omega_X & \delta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ITRF\ 2000}$$

$$T_1(t) = T_1(t_0) + (\frac{dT_1}{dt}) \cdot (t - t_0)$$

Computation of Coordinates in EUREF 89

- From coordinates in ITRFyy for epoch t_0
  - Compute ITRFyy coordinates at 1989.0
  - Transform into EUREF 89 coordinates
  - Compute velocity in EUREF 89
- From GPS measurements and fiducial stations in ITRFyy
  - Transform ITRFyy coordinates of fidudial stations referred to epoch t_0 into coordinates referred to central measurement epoch t_c
  - Carry out field measurements at epoch t_c
  - Process GPS measurements to obtain ITRFyy coordinates referred to epoch t_c
  - Convert to EUREF 89 coordinates at epoch t_c
  - Convert to EUREF 89 coordinates at epoch 89.0

Reference systems of GNSS (=ITRS)

- GPS
  - WGS 84, defined in 1987 using TRANSIT observations. **1-2 m**
  - WGS 84 (G730) defined in 1994, WGS 84 (G873) defined in 1996, agrees with **ITRF94** at **10cm** level
  - WGS 84 (G1150), defined in 2002, agrees with **ITRF2005** up to **1cm**
- GLONASS
  - PZ-90, defined in 1990 using 26 ground stations
  - Agrees with ITRF at **meter level** (from a joint campaign in 1999)
  - PZ-90.11 realized in 2013 is compatible with ITRF 2008
- Galileo
  - GTRF agrees with ITRF within **1.5cm**

Some remarks

- Achievable accuracy of latest ITRF: **1cm**, **1ppb**
- Faciliating transformation from national, triangulation-based coordinate systems to global geocentric GRS.
- Accurate geoid models to solve the problem:

$$2 + 1 \neq 3$$

$$(\varphi, \lambda) + (H) \neq (x,y,z)$$

GRS overview

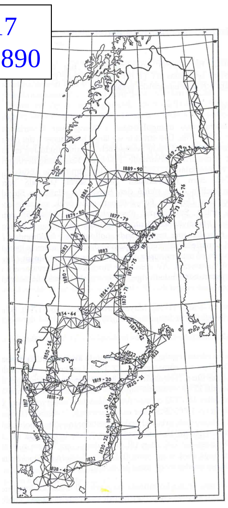
- Why? What is new?
- Astrogeodetic triangulation (2D)
- Height systems from levelling (1D)
- 3D geocentric reference systems
- Swedish geodetic infrastructure

Swedish Geodetic Infrastructure

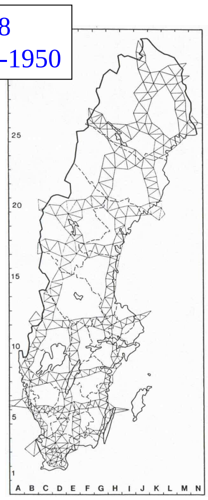
- Triangulation (2D) - RT90
- Precise levelling (1D) - RH 2000
- GNSS (3D) - SWEREF99
- Gravity - RG 82
- Geoid - SWEN 08LR
- Land uplift - RH 2000LU

National 1st-order Triangulation

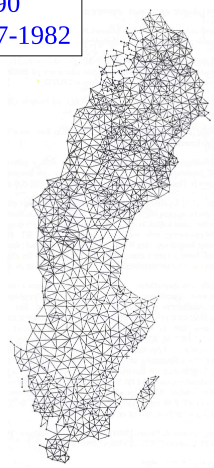
RT 1817
1815-1890



RT 38
1903-1950

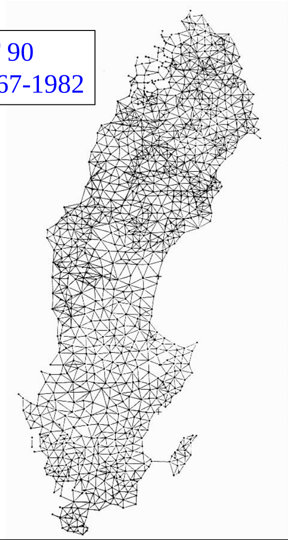


RT 90
1967-1982



3rd National Triangulation (RT 90)

RT 90
1967-1982



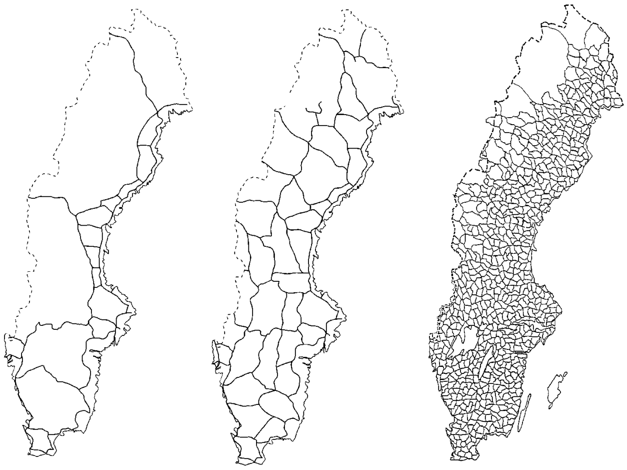
- Complete network with 15295 distances measured by **Geodimeters ®**, 5424 angle measurements, 366 first-order points/baselines/angles of RT 38,

Swedish national levelling networks

RH 00
1886-1905

RH 70
1951-1967

RH 2000
1978-2003



Swedish Motorized Levelling

3rd Swedish Precise Levelling (RH 2000)

- 50000 benchmarks, ~1 benchmark per km
- Field precision: **< 1 mm**/sqrt(km)
- Motorized levelling technology used
- New height reference system: **RH 2000**
- Consistent with **EVRS 2000**:
  - Normal height is used
  - Zero height at Amsterdam
  - Zero tidal system (i.e. Permanent deformation is retained, while permanent tidal effect is removed)

Swedish 3D reference frame

- SWEPOS
  - 21 permanent GPS stations (25 since March 2000)
  - Realization of Swedish 3D reference system (ETRF89)
  - Monitoring GNSS system integrity
  - Support network-RTK – VRS (70km, 1-2cm (H), 2-3cm (V))
- SWEREF 93
  - 4 Swedish GPS stations in EUREF 89 campaign
  - 23 GPS stations in DOSE campaign, computed in ITRF 91 at epoch 1993.6
  - 11 DOSE points (5 Swedish) are fitted to EUREF coordinates
  - The final set of coordinates is named **SWEREF 93**

3D reference frame SWEREF 99

- Based on a Nordic GPS campaign in 1999, with 49 stations incl. 25 SWEPOS stations
- Coordinates computed in ITRF97 for 1999.5
- Transformed to EUREF89 for 1989.0

- Adopted in 2000 as an realisation of ETRS 89
- Introduced in 2001 as Swedish national reference frame **SWEREF 99**

- Tectonic epoch: 1989.0
- Height epoch: 1999.5 (GPS campaign)
- Ellipsoid: GRS 1980

Transform SWEREF 99 to RT 90/RH 2000

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{RT\ 90} = \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} + (1+\delta s) \cdot R(\alpha_1, \alpha_2, \alpha_3) \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{SWEREF\ 99}$$

$$\delta x = -414.0979 \text{ metres}$$

$$\delta y = -41.3381 \text{ metres}$$

$$\delta z = -603.0627 \text{ metres}$$

$$\delta s = +0.000\,000\,000\,0 \cdot 10^{-6}$$

$$\alpha_1 = -0.855\,043\,431\,4''$$

$$\alpha_2 = +2.141\,346\,518\,5''$$

$$\alpha_3 = -7.022\,720\,951\,6''$$

Helmert model of 3D transformation

$$\begin{bmatrix} X_i^{(2)} \\ Y_i^{(2)} \\ Z_i^{(2)} \end{bmatrix} = \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} + s \cdot R(\alpha_1, \alpha_2, \alpha_3) \cdot \begin{bmatrix} X_i^{(1)} \\ Y_i^{(1)} \\ Z_i^{(1)} \end{bmatrix}$$

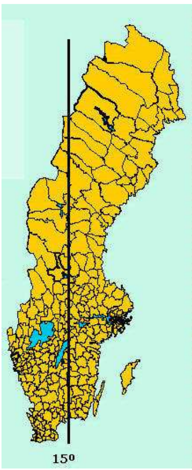
$$s = 1 + \delta s$$

$$\underset{3\times 3}{R} = R(\alpha_1, \alpha_2, \alpha_3) = R_3(\alpha_3) \cdot R_2(\alpha_2) \cdot R_1(\alpha_1)$$

$$= \begin{bmatrix} \cos\alpha_3 & \sin\alpha_3 & 0 \\ -\sin\alpha_3 & \cos\alpha_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\alpha_2 & 0 & -\sin\alpha_2 \\ 0 & 1 & 0 \\ \sin\alpha_2 & 0 & \cos\alpha_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_1 & \sin\alpha_1 \\ 0 & -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix}$$

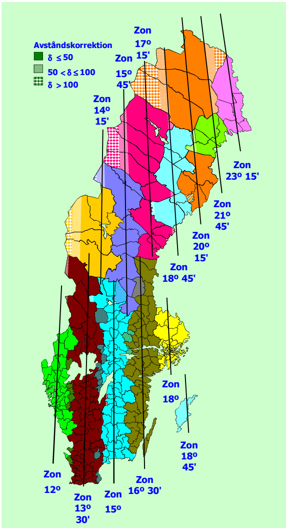
$$= \begin{bmatrix} \cos\alpha_2\cos\alpha_3 & \cos\alpha_1\sin\alpha_3 + \sin\alpha_1\sin\alpha_2\cos\alpha_3 & \sin\alpha_1\sin\alpha_3 - \cos\alpha_1\sin\alpha_2\cos\alpha_3 \\ -\cos\alpha_2\sin\alpha_3 & \cos\alpha_1\cos\alpha_3 - \sin\alpha_1\sin\alpha_2\sin\alpha_3 & \sin\alpha_1\cos\alpha_3 + \cos\alpha_1\sin\alpha_2\sin\alpha_3 \\ \sin\alpha_2 & -\sin\alpha_1\cos\alpha_2 & \cos\alpha_1\cos\alpha_2 \end{bmatrix}$$

National Projection - SWEREF 99 TM



- (x,y,z)-coordinates based on SWEREF 99
- Geodetic coordinates (φ,λ,h) referred to GRS 80 reference ellipsoid
- Planar coordinates (x,y) in Gauss conformal projection, i.e Transversal Mercator projection
- Central meridian λ_0 = 15 °
- Scale reduction factor: k_0 =0.9996
- False easting: y-coordinates shifted by +500 km
- Whole Sweden in one UTM zone (UTM 33)

Regional projection zones

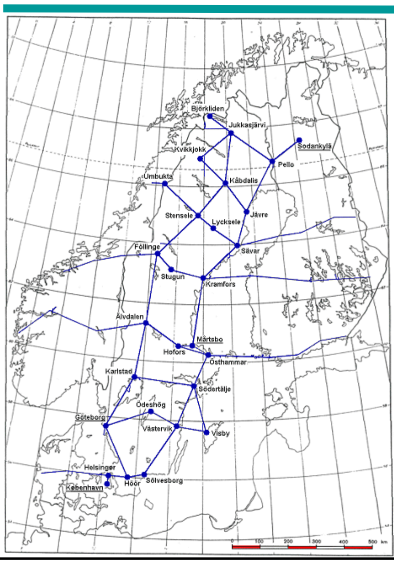


- 12 zones: **SWEREF 99 dd mm**
- 12 Central meridians:

12° 00'	**14° 15'**
13° 30'	**15° 45'**
15° 00'	**17° 15'**
16° 30'	**18° 45'**
18° 00'	**20° 15'**
	21° 45'
	23° 15'

- Scale reduction factor: $k_0 = 1$
- False easting: y + 150 km

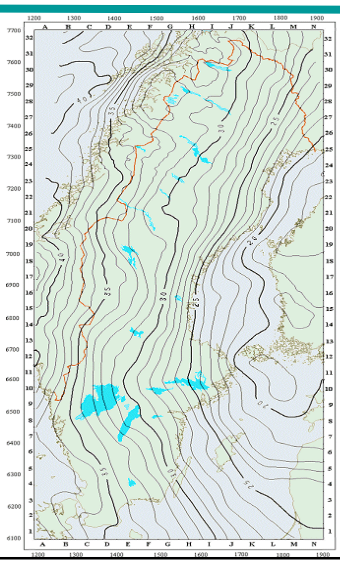
The 3rd zero-order gravity network - RG82



- RG 41, RG 62, RG 82
- 25 zero-order points measured 1981-82
- Relative gravity measurements, connected to other Nordic countris
- 4 absolute gravity points:
  - Mårtsbo, Göteborg, Copenhagen and Sodankylä
- Mårtsbo: **g = 981 923,484 mgal**
- Time epoch: 1982.0
- Zero tidal system

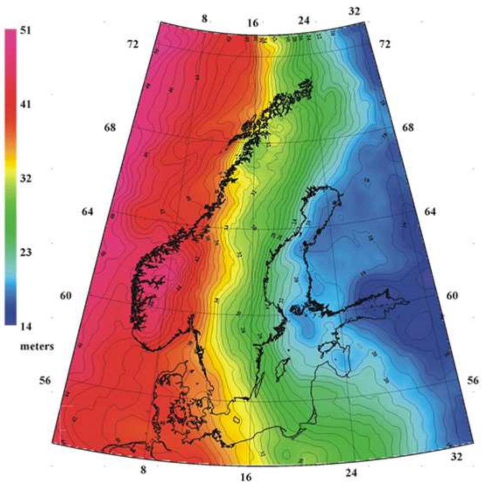
Swedish geoid models

SWEN 01L



- RN 92
  - NKG 89 geoid computed at KMS, using OSU89 and GRS 80
  - NKG 89 transformed to refer to RT 90
- SWEN 01L
  - NKG 96 computed using EGM 96
  - NKG 96 fitted to SWEREF 99 and RH 70 at 91 GPS/levelling points (4 cm rms, 15cm max. residuals)
  - Land uplift between 1970.0 and 1999.5 has been included
  - A height correction model

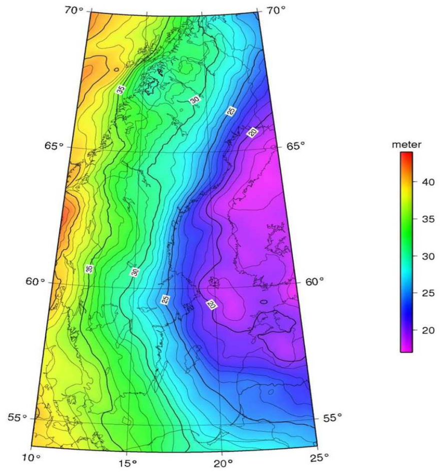
Gravimetric Geoid Model - SWEN 05LR



SWEN 05L**R**

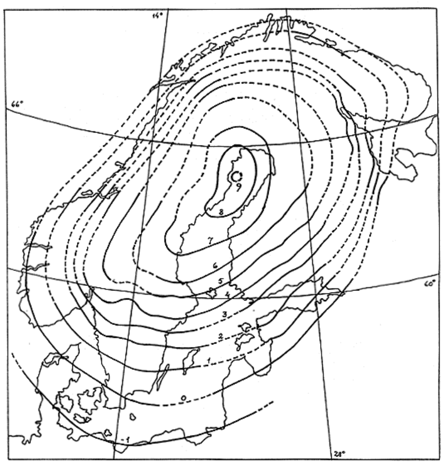
- NKG 2004 fitted to SWEREF 99 and RH 2000 at 1178 GPS/levelling points
- 0.5 years land uplift correction (between 1999.5 and 2000.0)
- Remaining residuals are smoothly interpolated
- A height transformation model from SWEREF 99 to RH 2000
- Accuracy:
  - 1.3 cm at ”RIX 95” GPS points
  - 3.7 cm other places
  - 5-10 cm in mountain areas

Gravimetric Geoid Model - SWEN 08LR

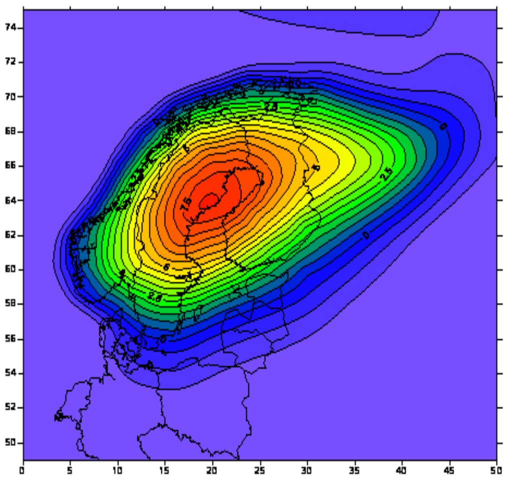


- Computed using the KTH method by modifying Stokes' formua and combining global gravitational models with terrestrial gravity data.
- Fitted to 1570 GPS/levelling points (SWEREFF 99 / RH 2000)
- Remaining residuals are smoothly interpolated
- A height transformation model from SWEREF 99 to RH 2000
- Accuracy:
  - 1 – 1.5 cm in most areas
  - 5 – 10 cm in Northwest without levelling
- **SWEN 15LR** to be introduced soon.

Swedish land uplift model (mm/year)



Martin Ekman (1998)



RH 2000 LU