High accuracy GNSS positioning

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GNSS – Global Navigation Satellite System

Global systems:
- GPS (American) – currently 31 satellites in orbit
- GLONASS (Russian) – currently 25 satellites in orbit
- Galileo (European) – currently 16 satellites in orbit
- Beidou (Chinese) – currently 18 satellites in orbit

Regional systems:
- IRNSS (Indian)

Augmentation systems such as:
- EGNOS (European)
- WAAS (American)
- QZSS (Japanese)

GNSS based positioning

GNSS positions can be determined with data from:

- Code observations only
  - More simple, faster, smaller and cheaper units
- Code and phase observations
  - More robust, better accuracy, but larger and more expensive units
- One frequency (L1 only)
  - More simple, faster, smaller and cheaper units
- Multiple frequencies
  - More robust, better accuracy, but larger and more expensive units
- GPS only
- GPS + GLONASS
- Other combinations of GNSS

=> Multi constellation multi frequency

Outline

- Carrier phase based positioning in review
- Relative positioning
  - Double differences, linear combinations, and ambiguity resolution
  - RTK – real time kinematic
- PPP – precise point positioning
  - Post processed PPP
  - Real time PPP
- Examples of applications

Satellite signals – GPS frequencies

L1: 1575.42 MHz
- Wave length: 19 cm
- Protected frequency

L2: 1227.60 MHz
- Wave length: 24 cm

L5: 1176.45 MHz
- Wave length: 25 cm
- Realized with 5 new block IIF satellites

Carrier phase observation equation

$$\Phi = \rho + d\rho + c\left(\delta_r - \delta^s\right) + \lambda N - I + T + \upsilon$$

Where:

Φ	= measured phase observation in metric unit
ρ	= geometric distance between receiver and satellite
$d\rho$	= orbit error
c	= speed of light
δ_r, δ^s	= receiver and satellite clock error
λ	= wave length
N	= ambiguity
I	= signal delay in ionosphere
T	= signal delay in troposphere
v	= multipath, receiver noise, antenna phase center variation etc.

Carrier phase based positioning methods

Classification of GNSS carrier phase based positioning methods

Post processing:

- Static – relative or precise point positioning
- Kinematic – relative or precise point positioning

Real time:

- Real time kinematic – relative
- Precise point positioning

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Single and double difference

Single difference between receivers:

$$\Phi_{AB}^{j} = \Phi_{B}^{j} - \Phi_{A}^{j} = \rho_{AB}^{j} + \lambda N_{AB}^{j} + c\delta_{AB}^{j}$$

Double difference:

$$\Phi_{AB}^{jk} = \Phi_{AB}^{k} - \Phi_{AB}^{j} = \rho_{AB}^{jk} + \lambda N_{AB}^{jk}$$

Observations from two satellites and two receivers, same epoch in time, used to generate two single differences, these are subtracted

=> elimination of both satellite and receiver clock errors, reduction of residual spatially correlated errors (mainly atmospheric effects), double differenced N is an integer

Linear combinations of observations

With dual frequency observations, linear combinations of the observations can be used in the positioning process

General formulation:

$$\Phi_{n,m}(i) = n\Phi_1(i) + m\Phi_2(i)$$

where:

- Φ are phase observations on L1 or L2
- n and m are constants
- i is the time epoch

Examples of L1 L2 linear combinations

Wide lane:

- n = 1 and m = -1
- Wave length of 86 cm

$$\Phi_{WL}(i) = \Phi_1(i) - \Phi_2(i)$$

Narrow lane

- n = 1 and m = 1
- Wave length of 11 cm

$$\Phi_{NL}(i) = \Phi_1(i) + \Phi_2(i)$$

WL often used as an intermediate step in N-resolution

WL and NL can be used in combination as constraint in positioning process (even or odd constraints)

Ionosphere free linear combinations

The first order ionospheric effect:

$$I = \frac{40.3 * TEC}{f^2}$$

where TEC is the Total Electron Content along the signal path. TEC is identical for L1 and L2

An ionosphere free (IF) combination of L1 and L2 observations eliminates this first order effect from the observation equations:

$$\Phi_{IF} = \Phi_{L1} - \frac{f_{L2}}{f_{L1}} \Phi_{L2} \qquad \Delta\nabla N_{IF} = \Delta\nabla N_{L1} - \frac{f_{L2}}{f_{L1}} \Delta\nabla N_{L2}$$

Linear combinations with more frequencies

Linear combinations of observations can also be carried out using e.g. the GPS L5 frequency, along with L1 and L2

Especially finding an optimum ionosphere free linear combination of the three frequencies is a research topic

Even more relevant with multi-GNSS

No de facto standard for multi-frequency linear combinations has yet been set

Ambiguity resolution (1)

$$\underline{y} = \underline{\underline{G}}\,\underline{\delta x} + \underline{\underline{A}}\,\underline{N} + \varepsilon$$

Where:

y	is observation vector, *measurement minus range*
G	is observation matrix (or the double difference matrix), contains double difference unit vectors
δx	is vector with corrections for the preliminary position (or element)
A	is unity matrix fitted to N
N	is double difference ambiguity vector
ε	is noise vector

(Notation: Misra and Enge, “Global Positioning System…”, 2006)

Ambiguity resolution (2)

The problem is rewritten to a minimisation problem in order to use least squares for resolution

$$c(\delta \underline{x}, \underline{N}) = \left\| \underline{y} - \underline{\underline{G}}\,\delta \underline{x} - \underline{\underline{A}}\,\underline{N} \right\|^2$$

Where:

- c is the cost function that we want to minimise
- c is the sum of the squares of the length of the residual vectors

Ambiguity resolution – float ambiguities

A direct solution to $c(\delta \underline{x}, \underline{N}) = \left\| \underline{y} - \underline{\underline{G}}\,\delta \underline{x} - \underline{\underline{A}}\,\underline{N} \right\|^2$

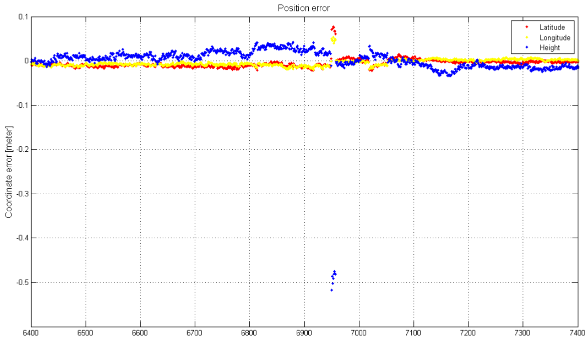
- will provide a reasonable position solution for short distances, when the noise level is low

This is a float resolution of the ambiguities - N will consist of float numbers (not integers)

=> The position solution will be unstable and inaccurate, and very sensitive to noise and changes in satellite geometry

Float ambiguities

- Plot shows position solution with fixed and float ambiguities
- Large coordinate deviation when ambiguities change from fixed to float



Ambiguity searching (1)

Search methods are introduced in order to find the most correct set of integer ambiguities

Common stepwise procedure:

1. Estimate preliminary position, e.g. from code observations
2. Define search area by accuracy of code solution
3. Search all possible combinations of N's (i.e. test all N's in a position solution and evaluate residuals)
4. Verify correct solution by statistical testing

Ambiguity searching (2)

Ambiguity searching can be very time consuming
Example:

- preliminary position known within 1 meter
- +/- 5 cycles in all directions
- 10 satellites observed

=> 10^{10} combinations to search

Several ambiguity searching methods to reduce processing time and increase reliability have been developed:

- Ambiguity Function Method (AFM), Fast Ambiguity Resolution Approach (FARA), Fast Ambiguity Search Filter (FASF), the LAMBDA method, and more

The LAMBDA method

Least Squares Ambiguity De-correlation Adjustment

The double differences are correlated, and with the LAMBDA method, de-correlation is used to reduce the search space

The method is developed at the Technical University of Delft in the Netherlands in the 1990'ies, and it is still considered state-of-the-art

Source code is freely available for non-commercial purposes

Steps in LAMBDA (1)

1. Least squares adjustment of all observations

$$c(\delta \underline{x}, \underline{N}) = \left\| \underline{y} - \underline{\underline{G}}\,\delta \underline{x} - \underline{\underline{A}}\,\underline{N} \right\|^2$$

=> float ambiguities and their covariance matrix

$$\begin{bmatrix} \hat{\delta x} \\ \hat{N} \end{bmatrix} \qquad Cov\begin{bmatrix} \hat{\delta x} \\ \hat{N} \end{bmatrix} = \begin{bmatrix} \Sigma_{\hat{\delta x}} & \Sigma_{\hat{\delta x}\hat{N}} \\ \Sigma_{\hat{\delta x}\hat{N}} & \Sigma_{\hat{N}} \end{bmatrix}$$

2. De-correlate the float ambiguities by Z transformation

$$\underline{\hat{M}} = \underline{\underline{Z}} \cdot \underline{\hat{N}}$$

Steps in LAMBDA (2)

3. Identify possible integer candidates, *M* of the transformed ambiguities, $\hat{M}$

4. Transform the integer candidates back using inv(**Z**) transformation

$$\underline{N} = \underline{\underline{Z}}^{-1} \cdot \underline{M}$$

5. Compute the coordinates using the integer values of ambiguities and validate the solution

$$\underline{y} = \underline{\underline{G}}\,\delta \underline{x} + \underline{\underline{A}}\,\underline{N} + \varepsilon$$

LAMBDA method - example

Reduction of the search space – example:

Known solution: $\mathbf{N} = (2,4)^T$

Float solution: $\hat{\mathbf{N}} = (3.875, 5.400)^T$

Z transform: $\mathbf{Z} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$

Transformed float ambiguities: $\hat{\mathbf{M}} = (-1.525, 9.975)^T$

Transformed fix ambiguities: $\mathbf{M} = (-2, 10)^T$

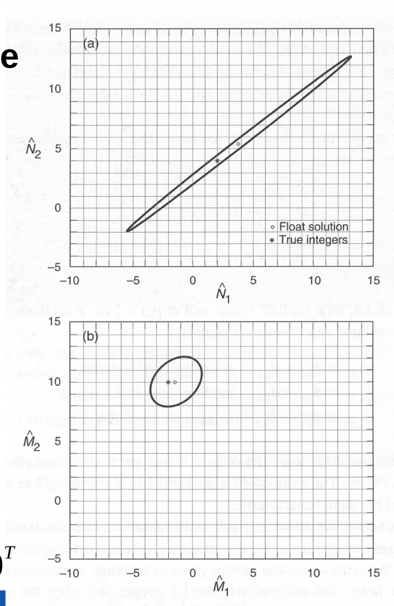


Illustration: Misra and Enge, “Global Positioning System…”, 2006

Ambiguity validation

When the ambiguities are resolved to integers, validation of the results is important

Because:

- The accuracy of a position based on ambiguities resolved to *wrong integers* can be *worse* than a position based on float ambiguities

The quality of a set of ambiguities depends on the number of observations, the geometry (DOP) and the chosen method

Ambiguity validation

The ratio test can for example be used:

$$\frac{\left(\mathbf{v}^T\mathbf{P}\mathbf{v}\right)_{nextbest}}{\left(\mathbf{v}^T\mathbf{P}\mathbf{v}\right)_{best}} > \text{critical value}$$

Where v is the residual vector and P is the weight matrix from the least squares adjustment

The “critical value” can be set from the number of degrees of freedom in the adjustment or set to a fixed value, e.g. 3

Ambiguity validation is also carried out using statistical testing, most commonly χ^2 testing, with other test variables

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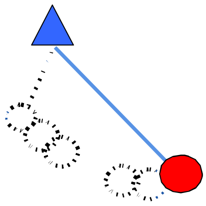
Real time kinematic (RTK)



Relative carrier phase based positioning in real time

Requires at least two GNSS receivers, one located in a known position

Requires data link for transfer of data between reference station and rover

- Most common is mobile internet, but also UHF radio is used

Position of rover determined in real time, normally using double differences, wide lane, and ambiguity search methods

- Obtainable position accuracy 1 – 5 cm

RTK – initialization

- During initialization, carrier phase data is transmitted from reference to rover, double differences are determined in the rover, and the ambiguities are resolved
  - Positioning normally possible with float ambiguities, but larger uncertainty in position solution
- After initialization, RTK corrections are transmitted to the rover, and the position is determined continuously or in a stop-and-go mode
- If cycle slip occur for one satellite, a new ambiguity is estimated for the given satellite while positioning is ongoing
- If contact to more satellites is lost, so four or less satellites are observed, all ambiguities are re-set and a new initialization must be carried out

Error sources and RTK

Illustration: www.leica-geosystems.com

With relative carried phase based positioning, like RTK, the effect of the spatially correlated errors:

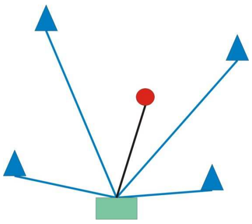
- Satellite clock error
- Satellite position error
- Atmospheric effects

is minimized

These errors are almost identical in reference and rover if the distance between the two receivers is short

Site dependent effects (multipath, signal blockage, antenna phase center variation etc.) are, however, not handled with RTK

Network RTK



RTK with a network of reference stations

Data from reference stations is combined in a common data processing in a control centre

The control centre computes corrections for the spatially correlated errors within the network

These corrections are then transmitted to the rover

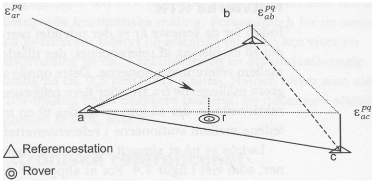
Distance to nearest reference station in the network can be increased up to e.g. 50 km

Network RTK

Basic principle:
Double differences between stations a, b and c in the network and the satellites p and q:

$$\Phi_{ab}^{pq}(t) = \rho_{ab}^{pq}(t) - \lambda N_{ab}^{pq}(t_0) + \varepsilon_{ab}^{pq}(t)$$

$$\Phi_{ac}^{pq}(t) = \rho_{ac}^{pq}(t) - \lambda N_{ac}^{pq}(t_0) + \varepsilon_{ac}^{pq}(t)$$



The value of the corrections, ε, are determined in a least squares adjustment – remember that ρ is known for the reference stations

ε contains residual effects of the spatially correlated errors as well as residual site dependent errors from the reference stations

Network RTK concepts

- Different concepts for estimation of network RTK corrections exist
- Common for the concepts is that ambiguities between reference stations in the network must first be estimated
- The concepts differ in the way error sources are handled
- Most important concepts today:
  - i-MAX originally developed by Leica Geosystems
  - VRS originally developed by Trimble
  - MAC originally from Leica Geosystems, further developed by GEO++

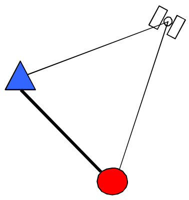
(Review based on Takac and Zelzer, 2008)

i-MAX concept

i-MAX - Individualised Master Auxiliary corrections

- Approximate position of rover transmitted to control center
- Data from a number of reference stations used for estimation of RTK corrections for the position of the rover
- Corrections used to correct raw GNSS data from one of the reference stations, selected as the ”master” station
- Corrected data from this reference station is transmitted to the rover

i-MAX concept



$$\tilde{\phi}_m^q = \phi_m^q + \delta\phi_{mr}^q$$

Where:

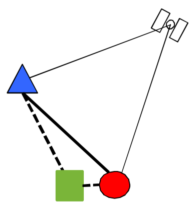
- $\tilde{\phi}_m^q$ is the observation from satellite q in reference station (the master) m, corrected for errors estimated for the rover position
- ϕ_m^q is the real observation from satellite q in reference station m
- $\delta\phi_{mr}^q$ is RTK error corrections for the single difference between reference station m, rover r, and satellite q

VRS concept

VRS - Virtual Reference Station

- Approximate position of rover transmitted to control center
- Data from a number of reference stations used for estimation of RTK corrections for the position of the rover
- Corrections used to generate artificial (or virtual) GNSS data as they would have been if a reference station was located right beside the rover
- This VRS data is transmitted to the rover

VRS concept



$$\tilde{\phi}_v^{\,q} = \tilde{\phi}_m^{\,q} + s_{m,v}^q + T_{m,v} + A_{m,v}^q$$

Where:

- $\tilde{\phi}_v^{\,q}$ is estimated VRS observation from satellite q in point v
- $\tilde{\phi}_m^{\,q}$ is the observation from satellite q in reference station m, corrected for errors estimated for the rover position
- $s^q_{m,v}$ is the single difference between reference station m, point v, and satellite q
- $T_{m,v}$ is the relative tropospheric effect between station m and point v
- $A^q_{m,v}$ is the antenna phase center correction for the single difference between station m, point v, and satellite q

MAC concept

MAC - Master Auxiliary Concept

- Data from all reference stations sent to the control center
- For each pair of reference stations, corrections for the observations are estimated
- Corrections are estimated between the chosen reference station (the master) against the other reference stations in the network (auxiliary stations)
- Corrections are divided into dispersive and non-dispersive parts
- Raw observations from the master are transmitted to the rover along with the corrections
- Advantage is much flexibility for MCMF and possibility to use one-way communication to the rover. Disadvantage is need for more bandwidth in the data communication link

MAC concept

$$\delta\phi^q_{mk,i} = S^q_{mk} - \phi^q_{mk,i} + \Delta t_{mk} + A^q_{mk,i} + \frac{c}{f_i} a^q_{mk,i}$$

Where:

$\delta\phi^q_{mk}$	is RTK error correction for the single difference between master station m, auxiliary station k, and satellite q
S^q_{mk}	is estimated geometric range for the single difference
$\phi^q_{mk,i}$	is the single difference of the raw observations between satellite q, master station m, and auxiliary station k
Δt_{mk}	is estimated receiver clock error
$A^q_{mk,i}$	is estimated antenna phase center correction
$a^q_{mk,i}$	is estimated integer ambiguity in units of cycles
f_i	is frequency (e.g. L1 or L2)
c	is speed of light

MAC concept

- In order to solve the ambiguities, double differences are introduced, so the term $a^{q}_{mk,i}$ is further developed
- The corrections, $\delta\phi^{q}_{mk}$ are further factorised into:
  - $\delta\phi^{q}_{mk,\gamma}$ dispersive (frequency dependent) part
  - $\delta\phi^{q}_{mk,\chi}$ non-dispersive part
- In the rover, the raw observations from the master station are used together with a “choice” of the corrections
- Note the flexibility in applying more frequencies, and different approaches for estimating the corrections

The RTCM format

- RTCM – Radio Technical Commission for Maritime standards
- Proprietary formats for transmission of RTK corrections to users have been used extensively
- For compatibility across receiver brands, the RTCM data format has become a de facto standard
- The RTCM format is undergoing a major modernisation to meet the needs of RTK (and PPP) positioning with multiple constellations and multiple frequencies

RTCM state space representation (SSR)

In classical RTK, corrections are generated in the observation space, i.e. corrections to the observations, and many error sources are lumped into combined corrections

With SSR, more error sources are estimated separately and with different temporal update rates based on physical models:

- Orbit corrections, different models for different GNSS
- Satellite clock correction terms
- Satellite code and phase biases
- Ionosphere VTEC values
- Troposphere signal delay

Ongoing RTK related research

RTK is a well known operational technique, but there are still challenges and ongoing research

For instance; integrity in network RTK:

- At system level: How to identify and eliminate errors in RTK corrections before they are transmitted?
- At rover level: How to (better) identify and eliminate position errors?

Possible solutions: Take advantage of the new GNSS (Galileo and Beidou), time series analyses and (Kalman) filtering etc.

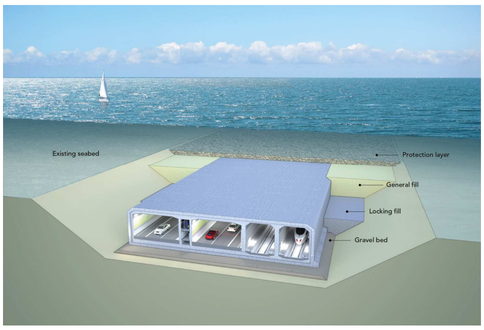
Example – the FFL positioning system

The following components are needed in order to establish a network RTK service

- Geodetic infrastructure
    - Geodetic reference frame and possibly geoid model
- Stable GNSS reference stations
- Data communication between reference stations and control center
- Software platform for data processing
- Data communication from control center to RTK users

Illustrated with an example of the positioning system for the Fehmarnbelt Fixed Link between Germany and Denmark

Case: Positioning system for the Fehmarn Fixed Link



Figures from www.femern.com

Fehmarn Fixed Link – GNSS stations

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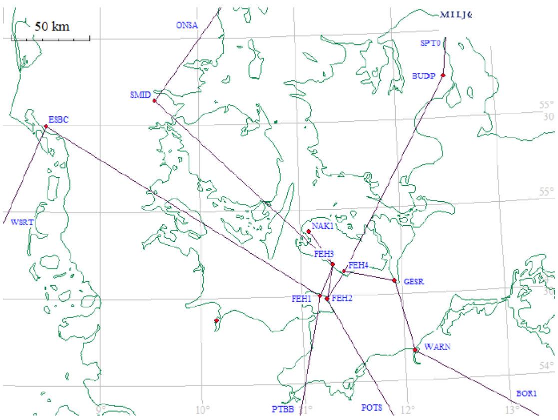
Case: Reference frame for the Fehmarn Fixed Link

Coordinates for the four permanent stations was determined with seven days of GPS-data and the Bernese software by the Danish Geodata Agency

Reference frame: ITRF2005



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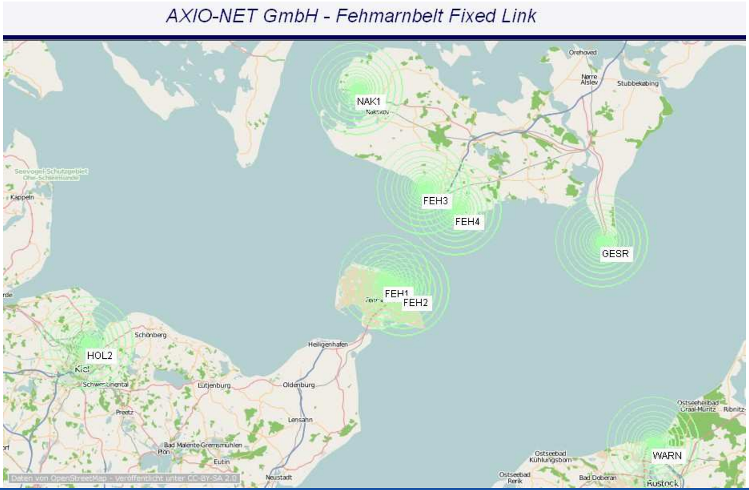


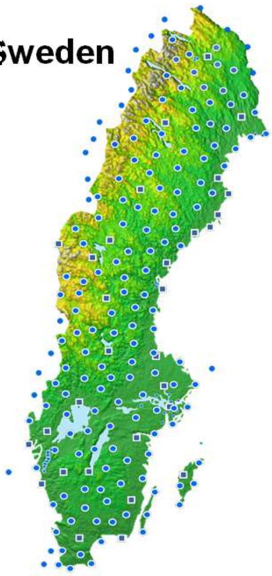
FFL GNSS RTK-service

- Network RTK service
- Based on the GEO++ software platform
- Both VRS and MAC data transmitted
- Operated by AXIO-NET GmbH in Germany for Femern A/S in Denmark

FFL RTK service, redundancy

- A network RTK service with single station RTK as backup
- Three different methods for data communication between GNSS stations and control center
- Two redundant control centers at two different addresses in Germany
- RTK-data transmitted using the RTCM data format both via UHF radio and via mobile internet using the NTRIP protocol
  - UHF radio provides only MAC data, because VRS require two-way communication with users

National services – example Sweden



Nationwide service

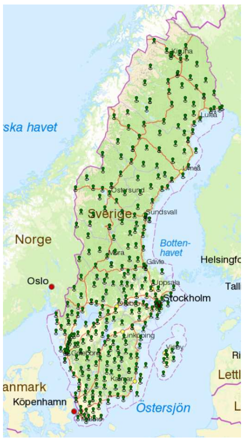
Operated by Lantmäteriet in Gävle

Based on the SWEPOS network of permanent stations

VRS-based service

Illustration from http://swepos.lmv.lm.se/

SWEPOS examples of differential services



SWEPOS, Swedish system, around 350 permanent GNSS stations

Operated by the National Land Survey

Services: network RTK, DGPS, raw data, post-processing

Phase and code corrections for GPS and GLONASS are distributed via mobile internet

Permanent GNSS stations

Photos from Lantmäteriet and Danish Geodata Agency

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Precise Point Positioning (PPP)

Precise Point Positioning (PPP) is high accuracy positioning with only **one** carrier phase GNSS receiver (non-differential approach)

Post processed PPP is used in applications where:

- Distance to a reference station is large e.g. off-shore
- Correlation between receivers is not wanted e.g. geodynamic monitoring

Real time PPP is used in applications where:

- Distance to a reference station is large e.g. in remote areas (mountains, desserts etc.) and offshore
- RTK could also be used but there is no RTK service, e.g. precision farming on the North American plains

Precise Point Positioning (PPP)

PPP is in principle based on a network of reference stations, but a much sparser network than needed for network RTK

Data from the reference stations is used for estimation of precise satellite orbits and clock errors

Dual frequency receivers must be used to account for first order ionospheric effects

Tropospheric effect is estimated in the positioning process

Models for earth tide, ocean loading, antenna phase center variation, differential biases in receivers etc. must be applied

Accuracy at cm-level can be obtained after convergence

Observation equation for phase and code

$$\Phi_f = \rho + d\rho + c\left(\delta_r - \delta^s + b_{r,f} - b_f^s\right) + \lambda_f N_f - I_f + T + \upsilon_f$$

Same as before (slide no. 5) but the following is added:

b_r, b^s = receiver and satellite phase bias

f = sub notation for frequency

$$P_f = \rho + d\rho + c\left(\delta_r - \delta^s + B_{r,f} - B_f^s\right) + I_f + T + \varepsilon_f$$

P = pseudorange observation

B_r, B^s = receiver and satellite code bias

ε = code noise, multipath, etc.

Hardware biases in PPP

Code and phase hardware biases are small time delays between transmission (or reception) of signals on the different frequencies

Code and phases biases cancel out in relative positioning (double differences), but must be accounted for in PPP by:

- Estimation or elimination in the positioning process
- Adding externally estimated corrections into positioning process

Inter-system biases must also be accounted for when merging more GNSS systems, and inter-frequency biases are important to consider when working with GLONASS

Ambiguities in PPP

Classical PPP is with ionosphere free float ambiguities, long convergence time, and (Kalman) filtering.

Alternatively, the Melbourne-Wübbena, MW, linear combination of dual frequency code and phase observations can be applied:

$$MW = \Phi_{WL} - P_{NL} = -\lambda_{WL} N_{WL} + \left(\beta_{r,MW} - \beta^{s}_{MW}\right) + \varepsilon_{MW}$$

where WL is wide lane, NL is narrow lane, and the β's are functions of code and phase biases on L1 and L2

The wide lane ambiguity here can be resolved performing "double differencing" between observations. But result is very noisy, because noise is amplified in the linear combinations

PPP accuracy and convergence time

Obtainable position accuracy and convergence time in standard PPP – static mode

Table 1 Recommended convergence time for static PPP solution to converge (Seepersad and Bisnath 2013)

Horizontal Accuracy (cm)	Recommended convergence period
20	35 min
10	50 min
5	60 min
2	9 h
1	23 h
0.5	24 h

Ambiguities in PPP

Ambiguities are difficult to resolve because they can not be separated from the hardware biases as in relative positioning

Recently, some methods have emerged relying on e.g. decoupling of the clock errors or constraining the ionosphere error, but improvement in positioning performance is not so convincing

Combination of PPP and network RTK has also evolved using ”local” corrections of e.g. troposphere and ionosphere

- High density network of reference stations required to improve performance

Triple frequency and multi-GNSS solutions improve both convergence time and position accuracy

PPP accuracy and convergence time

Table below from Choy, Bisnath and Rizos (2017).

Convergence time to reach and stay at 20 cm. Position accuracy is given by RMS over seven days

Station	GPS	GPS + GLO	GPS + BDS	GPS + GLO + BDS
BNLA				
Time (<20 cm)	85 min	68 min	65 min	45 min
E (cm)	3.1	3.9	2.1	3.1
N (cm)	3.3	2.1	2.5	2.0
U (cm)	7.5	7.2	5.6	5.3

Adding both GLONASS and Beidou to GPS reduces convergence time and RMS

Compatibility issues in PPP

Important that the user is using same convention as service provider.

For instance:

- The IGS use P1 and P2 observations in estimation of their products e.g. differential satellite code bias (DCB)
- If the user is using C1 and P2, then a correction factor must be applied in order to utilize the DCBs estimated by the IGS

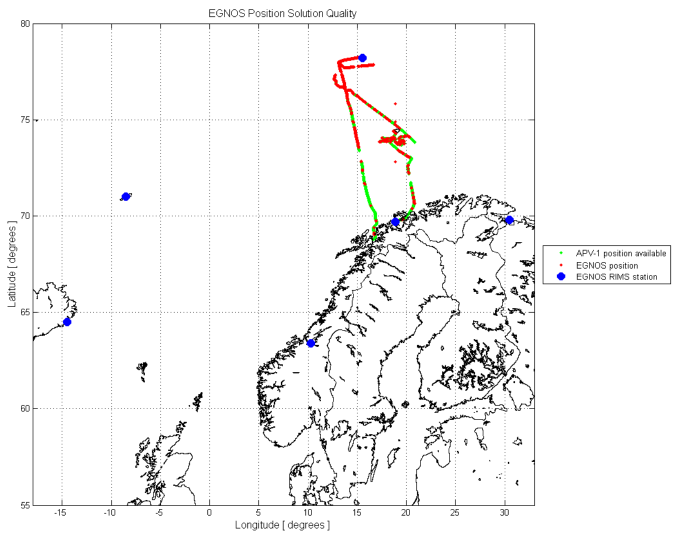
PPP – post processing

Final precise satellite orbits and clocks for instance from CODE or the IGS are applied

- PPP positions are determined using software from universities or commercial software, **examples**:
  - o The Gipsy software is widely used for geodynamic applications like time series analyses of permanent GNSS stations to monitor crustal motion or land uplift
  - o Terrapos software widely used for kinematic PPP applications, mainly airborne remote sensing applications like photogrammetry, gravimetry, laser scanning

- Automated web service
  - o Examples: NRCAN, SOPAC etc.

Example of the use of post processed PPP



Plot shown with permission from the Norwegian Mapping Authority

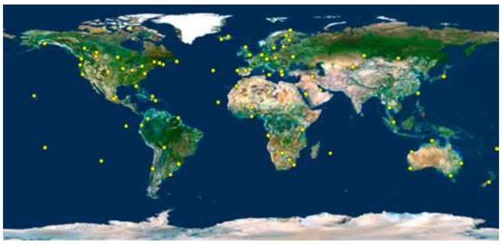
Precise Point Positioning (PPP) – real time

- PPP corrections are transmitted to users via internet or geostationary satellites, and the position is calculated in the rover receiver
- Position accuracy at the cm-level, after convergence
- A convergence (or initialization) time of 20-30 minutes is needed for the position solution to converge to cm-level accuracy

- IGS is streaming data for real time PPP freely available on the internet. Free software for positioning is also available e.g. RTKlib and BNC
- Global commercial services using geostationary satellites for data transmission offered by Fugro, Trimble, Veripos etc.

Trimble RTX – an example

Global network of permanent GNSS reference stations, all with Trimble receivers



Orbit and clock corrections as well corrections for atmospheric effects are estimated based on the reference stations and external data sources

Data transmitted to uses via geostationary Inmarsat-satellites

Galileo Commercial Service

The Galileo Commercial Service will be a PPP service

Plans and design not yet complete, but the purpose is to use the Galileo satellites for transmission of PPP-data

Compared to existing PPP services this is beneficial because current real time PPP services can only be used if internet or geostationary satellites area available, so such services are not available at high latitudes

Reference frames in PPP and RTK

PPP is carried out using a **global geodetic reference frame**; positions will be estimated in the same reference frame as orbits and clock corrections (e.g. the ITRF or the IGS reference frames) => globally consistent positions

RTK is normally carried out using a **national or regional reference frame**; positions will be estimated in the same reference frame as used for coordinates of the reference stations (e.g. ETRF or a national reference like SWEREF in Sweden) => nationally consistent positions

Geoid model needed for relation of heights to mean sea level or to local height reference system

Outline

- ✓ Carrier phase based positioning in review
- ✓ Relative positioning
  - ✓ Double differences, linear combinations, and ambiguity resolution
  - ✓ RTK – real time kinematic
- ✓ PPP – precise point positioning
  - ✓ Post processed PPP
  - ✓ Real time PPP
- Examples of applications

Applications of RTK and PPP

Both RTK and PPP can provide cm-level position accuracy in real time

Traditionally, real time cm-level accuracy was for land surveying and geodetic applications

Today these techniques are much more widely used

Figure: http://ciscokidz.com/index.htm

Autonomous driving

In autonomous driving, self driving cars, GNSS positioning is combined with other sensors such as vision, radar, INS etc.

Traditionally code based positioning for car navigation, but in autonomous driving lower uncertainty is required ⇒ need for RTK or PPP

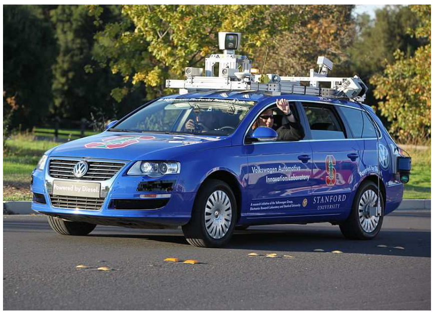


Image of test vehicle from Stanford University from wikipedia.com

Archaeology

RTK has been used in archaeology for some years

Provides new information on ancient ways of living when e.g. locations of remnants of old houses can be positioned at the cm level

In photo, single station RTK is used by KTH personnel in the Arctic, Spitsbergen, where the infrastructure is very sparse

Sports and training

Used for performance optimization of high level athletes, e.g. rowing and kayaking

Figure: www.gpsworld.com

Also for downhill skiing in e.g. Canada and Switzerland

Farming

RTK widely used for farming to optimize yield and reduce amount of fertilizer

Combination of RTK and real time PPP introduced in the USA

Pictures from Trimble.com and optisurface.ning.com

Mining

Used in open mining
for control
and guidance of
heavy machines

Pictures from:
www.worldhighways.com
and www.navipedia.net